Assignment 4

COMP 4500
Due: February 26, 2024 by 11:59 p.m.
Upload your Word, PDF, or LaTeX file(s) to Blackboard

Exercises from the Textbook (30 points):

Page 188: 29 (Hint: Consider sorting the degrees from largest to smallest.)

Page 246: 2, 3

Additional Exercises (70 points):

1. Consider the alphabet { ***t***, ***u***, ***v***, ***w***, ***x***, ***y***, ***z*** } with the following letter frequencies in a given text:
   t: 0.05, ***u***: 0.11, ***v***: 0.07, ***w***: 0.22, ***x***: 0.27, ***y***: 0.08, ***z***: 0.20

   Create an optimal Huffman encoding of this alphabet out of the symbols 0 and 1. Draw the tree you construct to create the encoding.

2. Count the number of inversions in the following list:

   3, 4, 5, 2, 7, 6, 1, 8

3. Use the Master Theorem to determine a Big Theta bound on the following recursive definition:

   $T(1) = 1$
   $T(n) = 10T(n/3) + 9n^3 \log n$

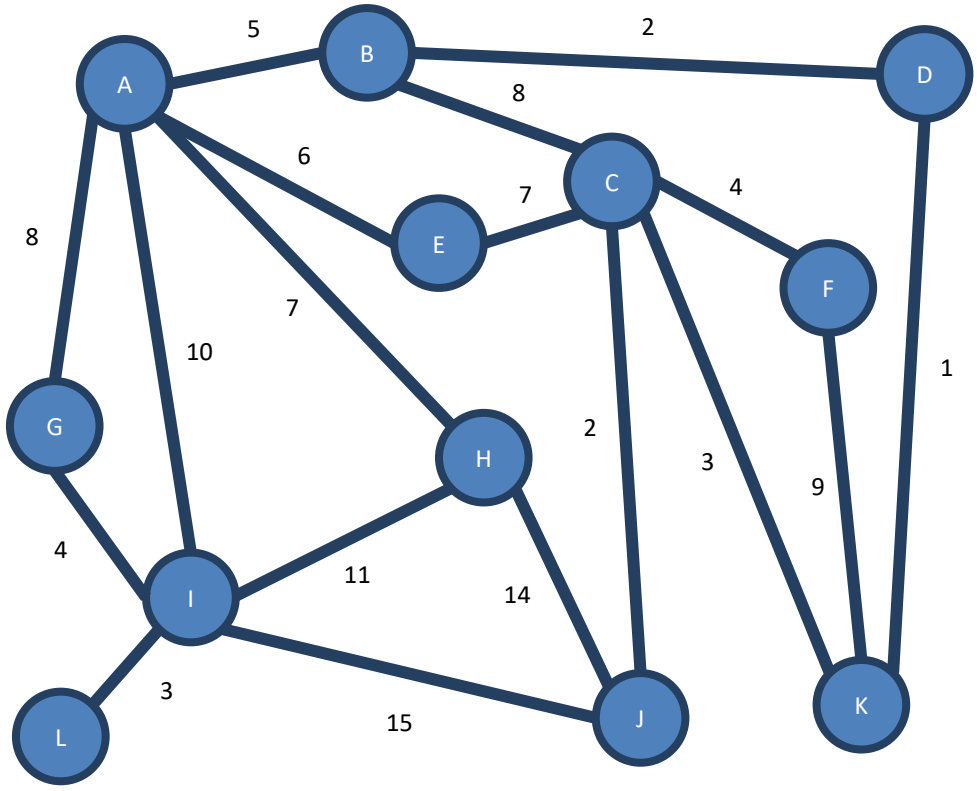
4. Use the Master Theorem to determine a Big Theta bound on the following recursive definition:

   $T(1) = 1$
   $T(n) = 8T(n/2) + 10n^2 \log n$

5. Use the Master Theorem to determine a Big Theta bound on the following recursive definition:

   $T(1) = 1$
   $T(n) = 16T(n/4) + 6n^2$

6. Use the clustering algorithm in the book to divide the following weighted graph into a clustering of maximum spacing with 3 clusters. Note that, when no edge is specified between two nodes, the distance between them is the lowest cost path between them.



7. Consider the following Java implementation of a Tower of Hanoi solving method:

```
public static void hanoi(int n, int a, int c, int b) {
   if(n == 1)
         System.out.println("Move the plate from " + a +
               " to " + c);
   else {
         hanoi(n - 1, a, b, c);
         System.out.println("Move the plate from " + a +
               " to " + c);
         hanoi(n - 1, b, c, a);
   }
}
```

Write a recurrence relation $T(n)$ that describes the running time of method **`hanoi()`**.

Then, use mathematical induction to prove an upper bound on the definition $T(n)$ that you came up with. If your definition of $T(n)$ is correct, it should **not** be possible to apply the Master Theorem to it.