Assignment 2

COMP 4290

Due: September 29, 2023

1. Consider the following vectors:

$$\begin{bmatrix} \frac{-2\sqrt{2}}{3} \\ \frac{-\sqrt{2}}{6} \\ \frac{\sqrt{2}}{6} \end{bmatrix}, \quad \begin{bmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

   Do these vectors constitute an orthogonal basis? Why or why not?

2. Consider the vectors from Question 1. Do those vectors constitute an orthonormal basis? Why or why not?

3. Compute the cross product $\mathbf{u} \times \mathbf{v}$ of the vectors defined below.

$$\mathbf{u} = \begin{bmatrix} 1 \\ -\sqrt{2} \\ 4 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 3 \\ 0 \\ \sqrt{3} \end{bmatrix}$$

4. Compute the matrix product **MN** where **M** and **N** are defined below.

$$\mathbf{M} = \begin{bmatrix} 1 & -2 & -3 \\ -1 & 0 & 4 \\ -1 & 2 & -1 \end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix} 2 & 5 & -3 \\ 4 & -3 & 0 \\ -1 & 3 & 5 \end{bmatrix}$$

5. Find the determinant of the following matrix.

$$\begin{bmatrix} 0 & 2 & -1 \\ 1 & -3 & -4 \\ -3 & -3 & \sqrt{5} \end{bmatrix}$$

6. Find the adjoint of the following matrix.

$$\begin{bmatrix} -2 & -1 & 1 \\ -2 & -1 & 4 \\ 1 & -3 & 0 \end{bmatrix}$$

7. Give the equation of a line in vector form (including its constants, of course) that passes through the points defined by the following vectors.

$$\begin{bmatrix} 5 \\ -2 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} -9 \\ 2 \\ 4 \end{bmatrix}$$

8. Give a single matrix in homogeneous notation that performs a transform equivalent to rotating a point -π/6 radians around the *x*-axis, π/4 radians around the *z*-axis, and then π radians around the *y*-axis.

9. Consider the matrix **T** below that performs a rigid body transform on a point. Give a matrix that will perform the inverse transform.

$$\mathbf{T} = \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} & 2 \\ 0 & 1 & 0 & -4 \\ -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

10. Consider the unit quaternions $\hat{\mathbf{q}}$ and $\hat{\mathbf{r}}$ given below.

$$\hat{\mathbf{q}} = \frac{1}{6}\, i - \frac{1}{2} j + \frac{5}{6} k + \frac{1}{6}$$
$$\hat{\mathbf{r}} = \frac{2}{5}\, i - \frac{2}{5}\, j + \frac{4}{5} k - \frac{1}{5}$$

Compute the spherical linear interpolation between the two with *t* = 0.6, that is, 60% of the way from $\hat{\mathbf{q}}$ to $\hat{\mathbf{r}}$.