Assignment 2

COMP 2230
Due: February 9, 2026 by 11:59 p.m.
Upload your Word, PDF, or LaTeX file(s) to Brightspace

Exercises from the Textbook (70 points):

Section 5.1: 26, 28, 57

Section 5.2: 14, 16

Section 5.3: 9

Section 5.4: 2

Section 5.6: 4, 29

Section 5.7: 4, 20

Section 5.8: 6, 12

Section 5.9: 19

Additional Exercises (30 points):

1. The Fibonacci numbers can be defined as follows:

   $F_k = F_{k-1} + F_{k-2}, k \geq 2$
   $F_0 = 1$
   $F_1 = 1$

   Thus, the first few Fibonacci numbers are 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …

   The definition given is a form of recursive definition which we will discuss later, but most of you should be familiar with the Fibonacci numbers already. Use a proof by induction to show that for $n \geq 0$, F_{3n+2} is even.

2. Using the same definition for the Fibonacci numbers, use a proof by mathematical induction to prove that the following explicit formula (shown in the book in Example 5.8.4) is equivalent to the recursive definition.

$$F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{n+1} \text{ for } n \geq 0$$

3. Find the error in the following proof by induction.

Theorem: All flowers smell the same.

Proof (by induction): Let F be the set of all flowers. Let $smell(f)$ be the smell of a flower $f \in F$. We will prove that, for all possible sizes of subsets of F, all the flowers in those subsets smell the same. Induction will proceed based on the size n of subsets of F.

Basis case: $n = 1$

For the case of subsets of F containing exactly 1 flower, every flower in the subset smells the same.

Induction case: $n \leq k$

Assume that all flowers in subsets of size k or smaller smell the same.

We will show that, given this assumption, all flowers in subsets of size $k + 1$ must also smell the same. To do so, we take an arbitrary subset $X \subseteq F$ such that the size of X is $k + 1$. Now, take two distinct flowers from X, f_1 and f_2. Let $Y_1 = X - \{f_1\}$ and $Y_2 = X - \{f_2\}$. Y_1 and Y_2 are of size k, so, by the induction hypothesis, all of the flowers within these sets smell the same. Pick an arbitrary $x \in Y_1 \cap Y_2$. Clearly, $x \neq f_1$ and $x \neq f_2$. But, $smell(f_2) = smell(x)$ by the induction hypothesis on Y_1 (since $f_2 \in Y_1$, which all smells the same). Likewise, $smell(f_1) = smell(x)$ by the induction hypothesis on Y_2 (since $f_1 \in Y_2$, which all smells the same). By transitivity, $smell(f_1) = smell(f_2)$, thus all the flowers in X smell the same.

Since we have shown that all flowers smell the same for subsets of any size, we have shown that all flowers smell the same. ■