Assignment 1

COMP 2230
Due: January 23, 2026 by 11:59 p.m.
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Exercises (100 points):

1. Prove or disprove the following conclusion based on the given premises. Next to each step in your argument, cite the applicable rule of inference.

   Premises:
   $x \to y$
   $\sim(x \wedge z) \to w$
   $\sim y$

   Conclusion:
   w

2. Construct a complete truth table for the statement: $\sim p \vee q \to (r \leftrightarrow q) \wedge s$

3. Consider the statement: $\big((\sim p \vee z) \wedge (y \vee z)\big) \to \big((p \wedge y) \to (x \wedge \sim z)\big)$
   Find an assignment of values for each symbol so that the entire statement is **true**.

4. Imagine a system of ternary logic. Instead of the two values of true and false, there would be **yes**, **no**, and **maybe**. How many rows would be contained in a ternary truth table with n variables? How many different, distinct ternary truth tables would be possible for two variables? (In other words, how many possible two-operand operators could there be in a ternary logic system?)

5. Logical NAND (the negation of logical AND) can replace all other Boolean connectives, though the formulas may become more complex. Write formulas using only NAND that are equivalent to the following expressions. You may wish to use the symbol ↑, which is a common shorthand for NAND. Be sure to supply appropriate parentheses for disambiguation.

   a. $\sim p$
   b. $p \wedge q$
   c. $p \vee q$
   d. $p \oplus q$ (XOR)
   e. $p \to q$

6. Define appropriate sets and predicates and write statements of predicate logic corresponding to the following statements of English.
    a. Every murderer chooses a suitable poison.
    b. One day, we'll be together.
    c. Good herrings are just herrings, but good cigars are Cubans.
    d. Nothing attracts anything.
    e. Nothing is better than chocolate.

7. Prove or disprove the claim: If x is an even integer and y is an odd integer, then $x \cdot y$ is an even integer. If you make a proof, give a justification for every step.

8. Prove or disprove the claim: Every positive integer can be written $a^2 + b^2 + c^2$, where $a, b, c \in \mathbb{Z}$. If you make a proof, give a justification for every step. **Note:** a, b, and c are not necessarily different. One or more of a, b, or c can be 0.

    Examples:
    $1 = 1^2$
    $2 = 1^2 + 1^2$
    $5 = 1^2 + 2^2$

9. Use a proof by contradiction to prove that there are no positive integers that are solutions to the equation $x^2 - y^2 = 10$. **Hint:** Factor.

10. Use a proof by contradiction to prove that no rational number is a solution to the equation $x^5 + x^4 + x^3 + x^2 + 1 = 0$. **Hint:** After rewriting x as the division of integers, use cases to consider the possibilities that the integers are both even, both odd, or one of each.