

Assignment 2

COMP 2100

Due: September 13, 2024

1. Answer the following questions about O bounds, assuming that $f_1(n)$ is $O(g_1(n))$ and $f_2(n)$ is $O(g_2(n))$. Assume that all functions $f_1(n)$, $f_2(n)$, $g_1(n)$, and $g_2(n)$ are continuous and non-decreasing.
 - a. Prove that $f_1(n) + f_2(n)$ is $O(\max(g_1(n), g_2(n)))$
 - b. Find an example showing that the statement $f_1(n) - f_2(n)$ is $O(g_1(n) - g_2(n))$ is not always true. The functions $f_1(n) - f_2(n)$ and $g_1(n) - g_2(n)$ should be continuous and non-decreasing.
 - c. Find an example showing that the statement $f_1(n)/f_2(n)$ is $O(g_1(n)/g_2(n))$ is not always true. The functions $f_1(n)/f_2(n)$ and $g_1(n)/g_2(n)$ should be continuous and non-decreasing.

2. Find a closed-form version of the following function $f(n)$. A closed-form version doesn't have summation notation.

$$f(n) = \sum_{i=5}^n (4i + 1)$$

3. Find a Θ bound for the following `for` loops in Java in terms of n . Assume that all variables are of type `int`.

a.

```
for (count1 = 0, i = 1; i <= n; i *= 2) {
    for(j = 1; j <= n; j *= 3) {
        ++count1;
    }
}
```

b.

```
for (count2 = 0, i = 1; i <= n; ++i) {
    for (j = 1; j <= i; j += 2) {
        ++count2;
    }
}
```

c.

```
for (count3 = 0, i = 1; i <= n; i *= 2) {
    for (j = 1; j <= i; ++j) {
        ++count3;
    }
}
```

4. Compare the following pairs of functions in terms of asymptotic growth. In each case, say whether $f(n)$ is $O(g(n))$, $f(n)$ is $\Omega(g(n))$, or $f(n)$ is $\Theta(g(n))$. State only the single **best** relationship between $f(n)$ and $g(n)$. Use the facts discussed in the slides.

	$f(n)$	$g(n)$
a.	$16n \log n$	$58n \log(\log n)$
b.	$n^2 \log n$	$n^{\frac{5}{2}}$
c.	$4n \log n$	$15n \log(n^3) + n$