Towards solving two-dimensional adjoint QCD with a basis-function approach

2017

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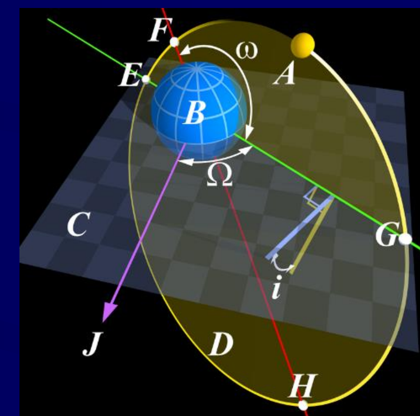

2019

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*Thanks to OSU for hospitality!

QCD_{2A} is a 2D theory of quarks in the adjoint representation coupled by non-dynamical gluon fields

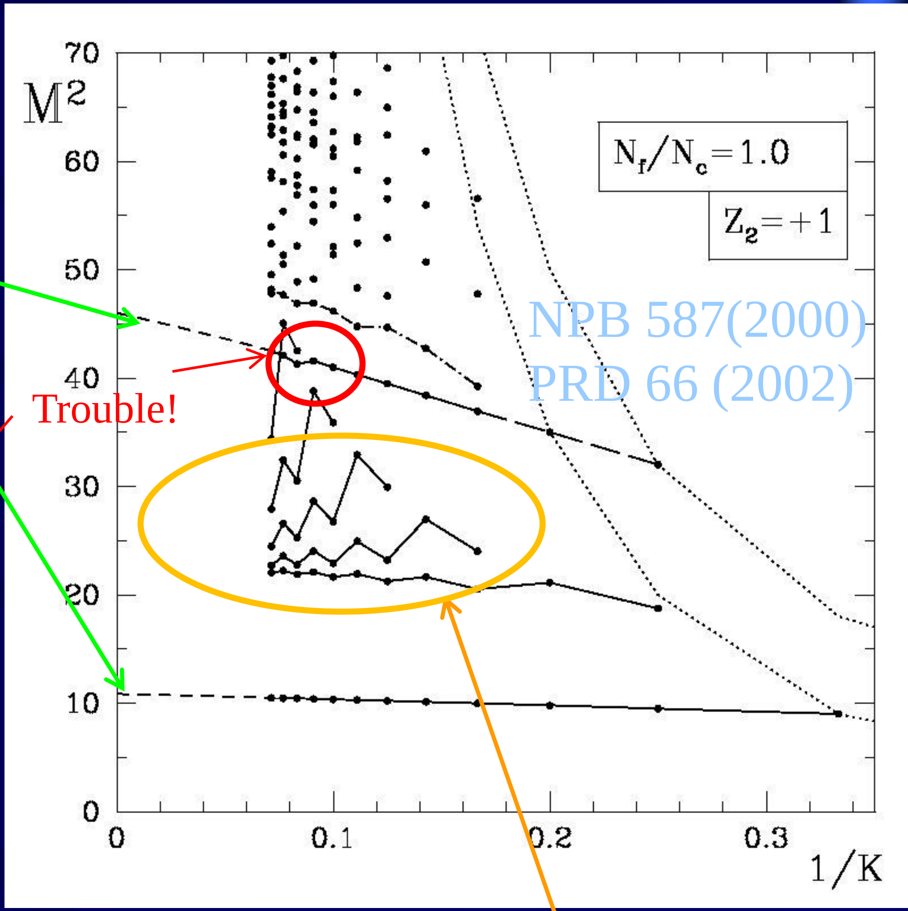
(“matrix quarks”)

- The Problem: all known approaches are cluttered with multi-particle states (MPS)
- We want “the” bound-states, i.e. single-particle states (SPS)
- Get also tensor products of these SPS with relative momentum
- SPS interact with MPS!

(kink in trajectory)

- DLCQ calculation shown, but typical (see Katz et al **JHEP 1405 (2014) 143**)

➔ Need to solve theory with new method → eLCQ



Group of approximate MPS

Algebraic Solution of the **Asymptotic** Theory I

- Since parton number violation is disallowed, the asymptotic theory splits into decoupled sectors of fixed parton number
- Wavefunctions are determined by 't Hooft-like integral equations $$M^2\phi_r(x_1,\ldots,x_r) = -\sum_{i=1}^{r}(-1)^{(r+1)(i+1)}\int_{-\infty}^{\infty}\frac{\phi_r(y,x_i+x_{i+1}-y,x_{i+2},\ldots,x_{i+r-1})}{(x_i-y)^2}dy,$$
- Need to fulfill "boundary conditions" (BCs)
  - Pseudo-cyclicity: $\phi_r(x_1,x_2,\ldots,x_r) = (-1)^{r+1}\phi_r(x_2,x_3\ldots,x_r,x_1)$
  - Hermiticity (if quarks are massive): $\phi_n(0,x_2,\ldots,x_n) = 0,$
- Use sinusoidal ansatz with correct number of excitation numbers: n_i ; $i = 1\ldots r$-1

$$|n_1,n_2,\ldots n_{r-1}\rangle \doteq \prod_{j}^{r-1} e^{i\pi n_j x_j} = \phi_r(x_1,x_2,\ldots,x_r)$$

PRD92: Algebraic Solution of the Asymptotic Theory (cont'd)

- $\phi_{3,sym}(x_1, x_2, x_3) = \phi_3(x_1,x_2,x_3) + \phi_3(x_2,x_3,x_1) + \phi_3(x_3,x_1,x_2)$
  $= \phi_3(n_1,n_2) + \phi_3(-n_2,n_1-n_2) + \phi_3(n_2-n_1,-n_1)$

  3 parton WF characterized by 2 excitation numbers

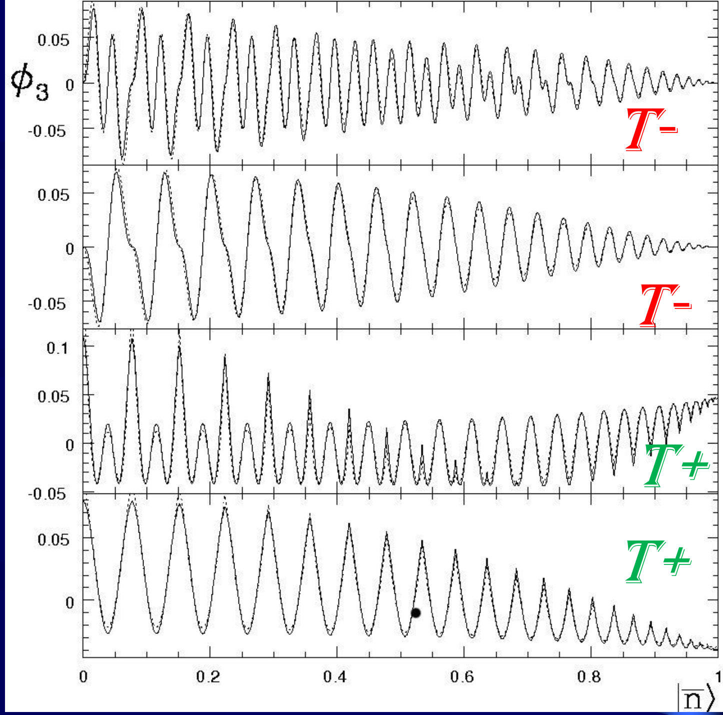
- $\phi_{4,sym}(x_1, x_2, x_3, x_4) = \phi_4(x_1,x_2,x_3,x_4) - \phi_4(x_2,x_3,x_4,x_1)$
  $+ \phi_4(x_3,x_4,x_1,x_2) - \phi_4(x_4,x_1,x_2,x_3)$

  $\mathcal{C}$: $(x_1, x_2, x_3, \ldots x_r) \rightarrow (x_2, x_3, \ldots x_r, x_1)$

- Therefore: $\phi_{r,sym}(n_i) \equiv \frac{1}{\sqrt{r}} \sum_{k=1}^{r} (-1)^{(r-1)(k-1)} \mathcal{C}^{k-1} \, \phi_r(n_i)$

  is an eigenfunction of the asymptotic Hamiltonian with eigenvalue $M^2 = g^2 N \pi^2 \sum_{k=1}^{r} |n_1^{(k-1)} - n_2^{(k-1)}|$

It's as simple as that and it works – up to point



- All follows from the two-parton ("single-particle") solution

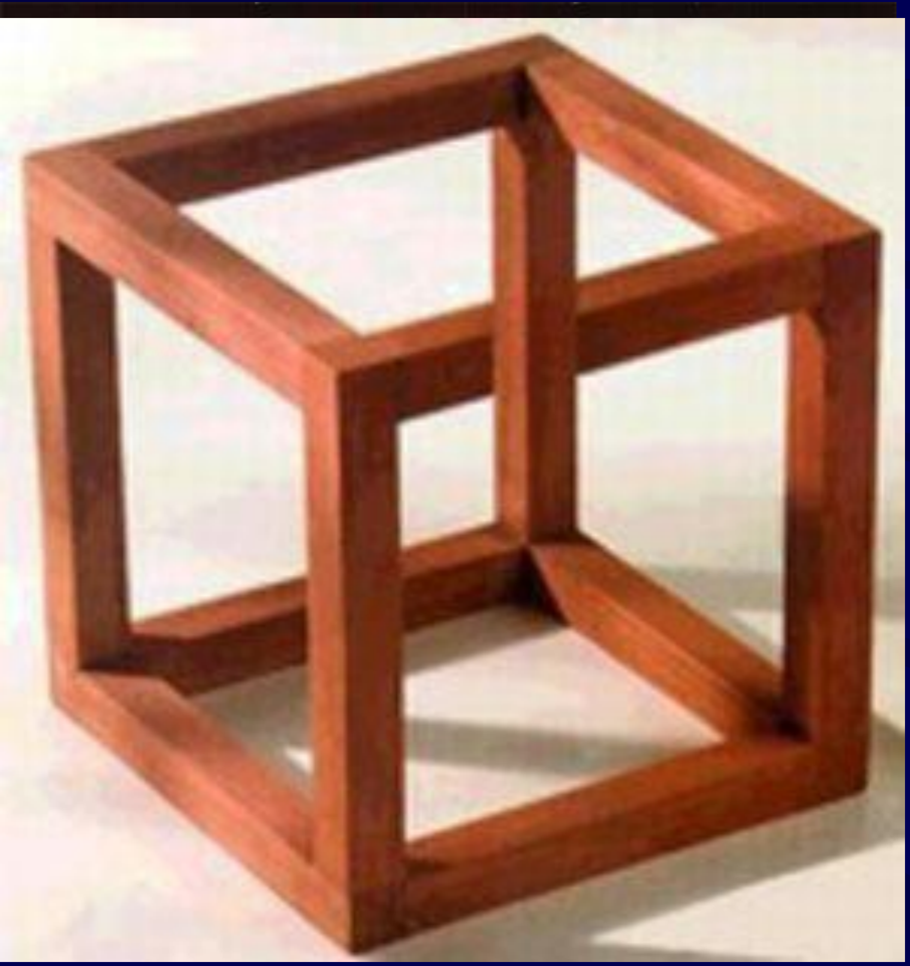
$$\frac{M^2}{g^2 N} e^{i\pi n x} = -\int_{-\infty}^{\infty} \frac{dy}{(x-y)^2} e^{i\pi n y} = \pi |n| e^{i\pi n x}$$

- Can clean things up with additional symmetrization:

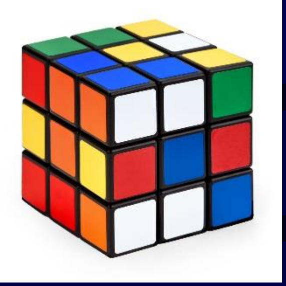
  T: $b_{ij} \rightarrow b_{ji}$

- Caveat: in higher parton sectors additional symmetrization is required (I said in 2015…)
- Want: EF should vanish if parton momenta vanish: $\phi(0, y, z, \ldots) = 0$
  - So at the boundary?
- How to achieve that? NOT with boundary conditions!
- This is not a boundary condition, but the behavior of the wavefunction on a hyperplane characterized by $x_i=0$

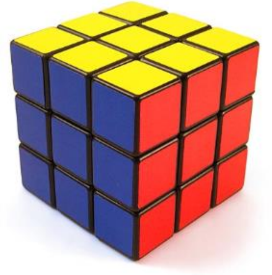
2017 – A New Hope



- Idea: symmetrize the wavefunction so it does what we want at $x_i = 0$
- But we need to keep it cyclic in the bulk!
- What if we can have the cake on one side – and eat from the other?
- **IS THIS IMPOSSIBLE?**



Possible, just need some group theory – and a group!



- What is the group, what is the symmetry?
- Want: EF should vanish if one or more parton momenta vanish: $\phi(0, y, z, \ldots) = 0$
- Have: modular ansatz, ie a bunch of terms with different excitation numbers or frequencies: $e^{i\pi\,(nx+my+..)}$
- Unsurprisingly: $e^{i\pi nx} - e^{-i\pi nx} = 0$ for $x = 0$
- Solution: add/subtract partner term with negative frequencies

The Devil is in the Details

- Must not screw up other symmetries
- Must have same mass eigenvalue
- Not impossible: construct lower-dimensional inversion, i.e. the transformation

$$S_i : |n_1, n_2, \ldots, n_i \ldots, n_{r-1}\rangle \rightarrow |-n_1, -n_2, \ldots, n_i - n_{i+1} - n_{i-1}(1-\delta_{1i}), \ldots, -n_{r-1}\rangle$$

…or rather permutation of frequencies, and therefore parton momenta, so that the modified frequency safeguards the mass eigenvalue

2018 – A New Symmetry

$$\mathcal{B} = \{1, \mathcal{C}, \mathcal{C}^2, \ldots \mathcal{C}^{r-1}, \mathcal{T}, \mathcal{T}\mathcal{C}, \ldots \mathcal{T}\mathcal{C}^{r-1}, \mathcal{I}, \mathcal{I}\mathcal{C} \ldots \mathcal{I}\mathcal{T}\mathcal{C}^{r-1}\}$$

$$\mathcal{E} = \{\mathcal{S}_1, \mathcal{S}_2, \ldots \mathcal{S}_{1/2(r-1)!-1}\}$$

- Every permutation is formally an automorphism and thus a symmetry
  - Subgroup $\mathcal{B}$ symmetrizes so that WFs are EFs of the Hamiltonian
  - Subset $\mathcal{E}$ symmetrizes so that they vanish or are max at $x_i=0$
- Construct a complete symmetrization under lower-dimensional inversion $S \in \mathcal{E}$ of the ansatz, but:
  - S operators do not commute
  - S operators do not commute with $\mathcal{T}, C \in \mathcal{B}$
  - Therefore left and right cosets of $\mathcal{B}$ are in general not the same: $S_i\mathcal{B} \neq \mathcal{B}S_i$

Solution: $\mathcal{G} = \mathcal{B} \times \mathcal{E}$

PRD96 (2018) 045011

$$\mathcal{E} = \{\mathcal{S}_1, \mathcal{S}_2, \ldots \mathcal{S}_{1/2(r-1)!-1}\}$$

$$\mathcal{B} = \{1, \mathcal{C}, \mathcal{C}^2, \ldots \mathcal{C}^{r-1}, \mathcal{T}, \mathcal{T}\mathcal{C}, \ldots \mathcal{T}\mathcal{C}^{r-1}, \mathcal{I}, \mathcal{I}\mathcal{C} \ldots \mathcal{I}\mathcal{T}\mathcal{C}^{r-1}\}$$

- Symmetrize until the group is exhausted!
  - "exhaustively-symmetrized Light-Cone Quantization" (eLCQ) ;-)
- The group of perturbations of r objects with inversions has a finite order: $|\mathcal{G}| = 2r!$
- Can show this explicitly by constructing group in *r* parton sector
- End result: Bona fide fully symmetrized states: |TIS; ***n*** > with quantum numbers under $\mathcal{T}$, $\mathcal{I}$, $\mathcal{S}$ and *r*-1 excitation numbers ***n***

Works! Massless Four-Parton Eigenfunctions

Numerical (solid) vs. Algebraic (dashed)

four states are in the massive theory ($\mu \neq 0$)

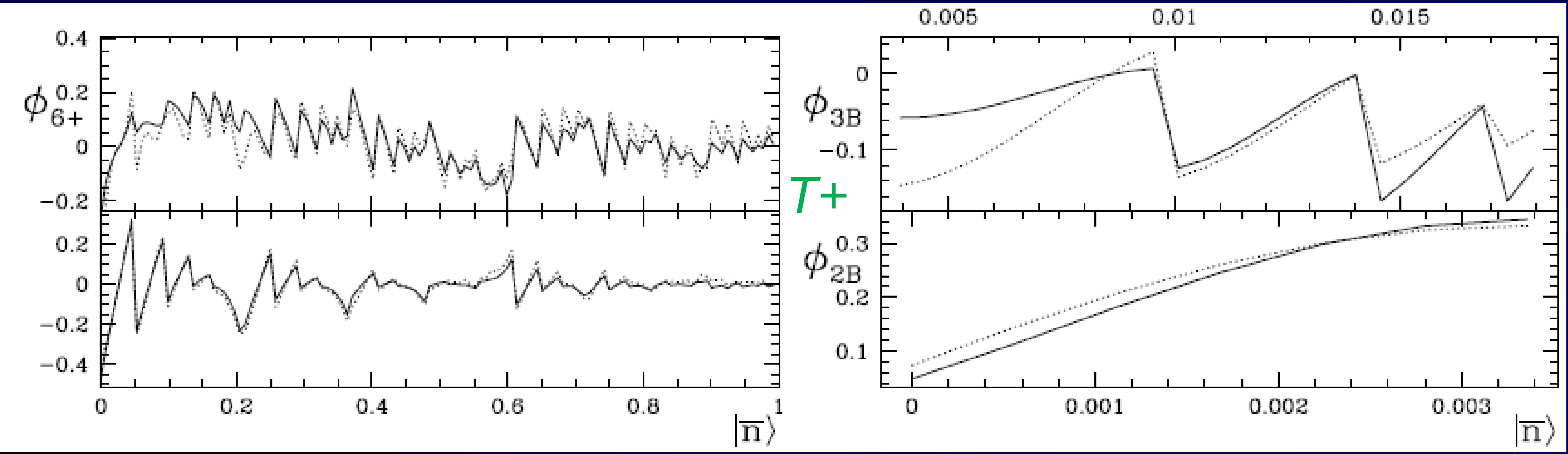
$$|1\rangle^{\mu\neq 0}_{+-+} = |4,-2,0\rangle_{12}, \qquad |1\rangle^{\mu\neq 0}_{-+-} = |4,0,2\rangle_{12},$$
$$|2\rangle^{\mu\neq 0}_{+-+} = |6,-2,0\rangle_{16}, \qquad |2\rangle^{\mu\neq 0}_{-+-} = |4,-2,0\rangle_{12},$$
$$|3\rangle^{\mu\neq 0}_{+-+} = \frac{1}{\sqrt{2}}\Big(|6,10,10\rangle_{20} + |8,10,6\rangle_{20}\Big), \qquad |3\rangle^{\mu\neq 0}_{-+-} = |6,4,6\rangle_{16},$$
$$|4\rangle^{\mu\neq 0}_{+-+} = |8,10,10\rangle_{20}, \qquad |4\rangle^{\mu\neq 0}_{-+-} = |6,-2,0\rangle_{16}.$$

In the massless theory they look like

$$|1\rangle^{\mu=0}_{+--} = |1,2,3\rangle_{6}, \qquad |1\rangle^{\mu=0}_{-++} = |1,0,1\rangle_{4}$$
$$|2\rangle^{\mu=0}_{+--} = |3,-2,-1\rangle_{10}, \qquad |2\rangle^{\mu=0}_{-++} = |1,-2,-1\rangle_{6}$$
$$|3\rangle^{\mu=0}_{+--} = |3,-2,-3\rangle_{12}, \qquad |3\rangle^{\mu=0}_{-++} = |3,0,1\rangle_{8}$$
$$|4\rangle^{\mu=0}_{+--} = |5,-2,-1\rangle_{14}, \qquad |4\rangle^{\mu=0}_{-++} = |3,-2,-1\rangle_{10},$$

Works! Six-Parton and Bosonic Eigenfunctions

Numerical (solid) vs. Algebraic (dashed)



6-parton fermionic theory
(adjoint fermions,
1440 terms in EFs)

Bosonized theory
(adjoint currents,
non-orthogonal basis)

Using the Asymptotic Basis – Approximating the Full Theory

- Expand the full EFs into a complete set of asymptotic EFs $$f_r(x_1, x_2, \ldots, x_r) = \sum_{\vec{n}} c_{r,\vec{n}} \phi_{r,\vec{n}}(x_1, x_2, \ldots, x_r),$$

- Project onto the asymptotic EFs to get an equation for the associated coefficient

$$M^2 \int d^r x \phi^*_{s,\vec{m}}(\vec{x}) f_r(\vec{x}) = M^2 \sum_{r,\vec{n}} \int d^r x c_{r,\vec{n}} \phi^*_{s,\vec{m}} \phi_{r,\vec{n}} = M^2 \sum_{r,\vec{n}} c_{r,\vec{n}} \delta_{s,r} \delta_{\vec{n},\vec{m}} = M^2 c_{s,\vec{m}}$$

- Problem: In adjoint QCD we cannot use Multhopp method of 't Hooft model → need to evaluate P.V. integrals numerically → Ongoing work

Conclusions/ Outlook

- Asymptotic theory was solved algebraically in all parton sectors → Coulomb (long range) problem solved!
- Can use complete set of solutions to solve full theory numerically with exponential convergence
- Can compute pair-production matrix elements $_{-+}\langle\bar{n}|P^-_{PV}|n,m,l\rangle_{+--}$ which look like $\int\int\frac{\sin\pi(n'x+m'y)}{(x+y)^2}dxdy$
- Can use eLCQ method to tackle other theories
  - Certainly with adjoint degrees of freedom
  - Possibly higher dimensions, since group structure seems independent of space-time symmetries

Thanks for your attention!

- Questions?

Not used

In other words, we need the Hamiltonian matrix elements with respect to the basis $\{\phi_{r,\vec{n}}\}$ of asymptotic eigenfunctions

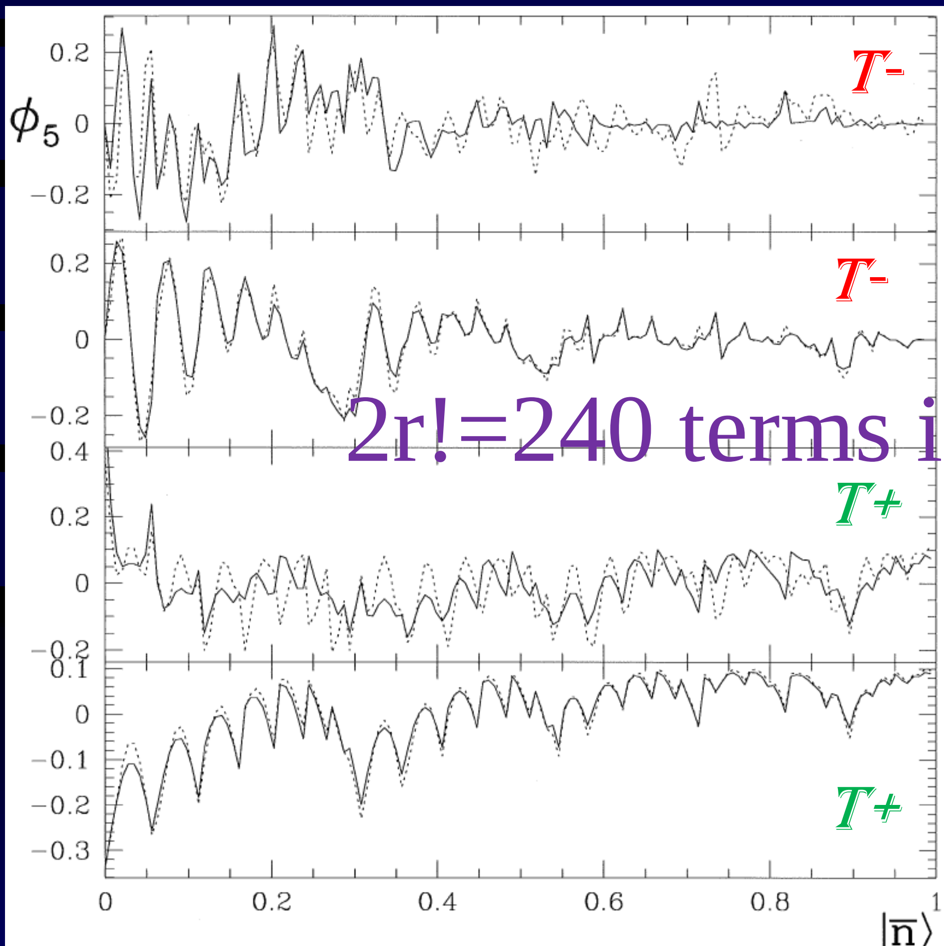
$$H_{\vec{m},\vec{n}} = \langle \phi_{r,\vec{m}} | \hat{H} | \phi_{r,\vec{n}} \rangle = \int_D d^r x' \phi_{r,\vec{m}}(\vec{x}') \int_0^{x_i+x_{i+1}} dy \frac{\phi_{r,\vec{n}}(y, x_i + x_{i+1} - y, \vec{\vec{x}})}{(x_i - y)^2},$$

where the $d^r x'$ integral is $r-1$ dimensional due to the $\sum x_i = 1$ constraint on the domain D, and the dy integral is one-dimensional.

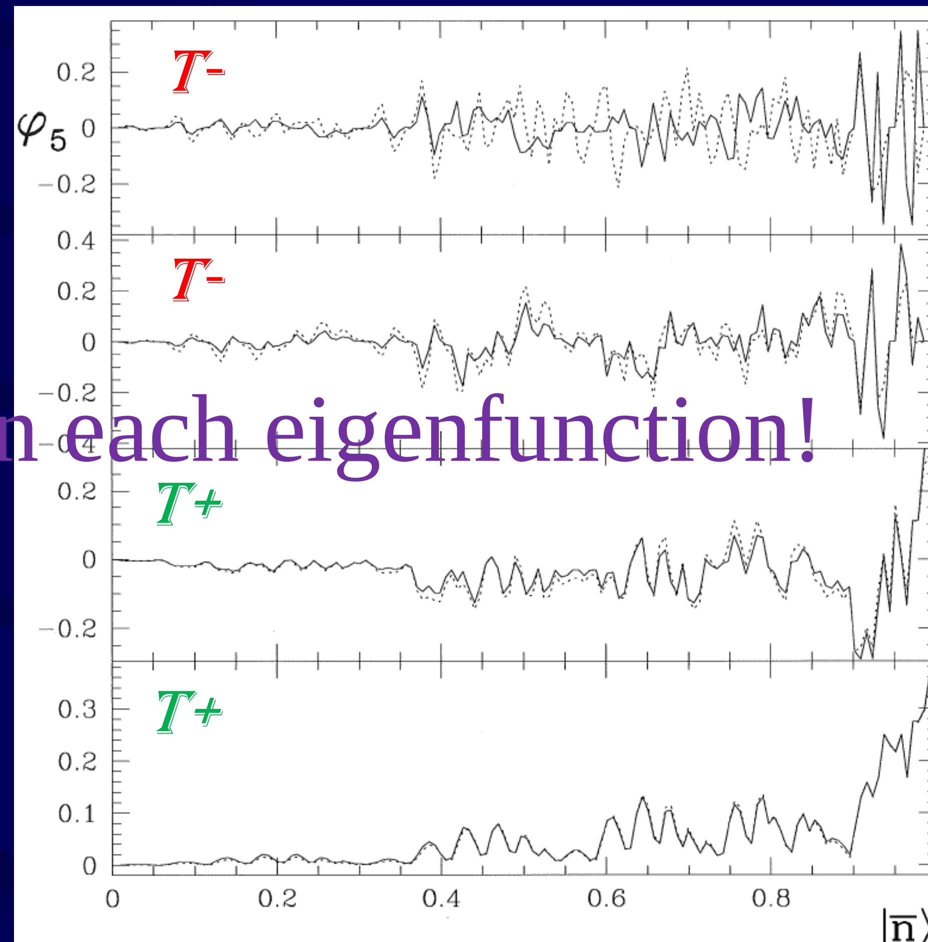
Works! Five-Parton Eigenfunctions

Numerical (solid) vs. Algebraic (dashed)

2r!=240 terms in each eigenfunction!



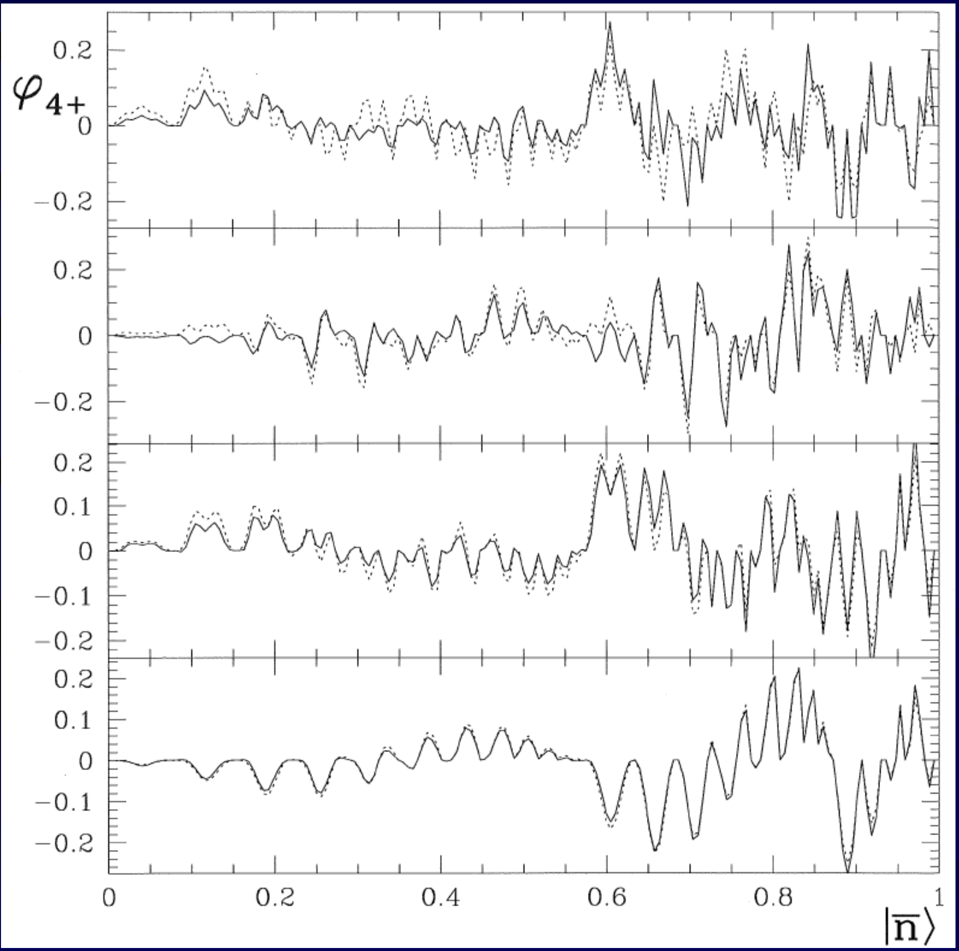
Massless theory

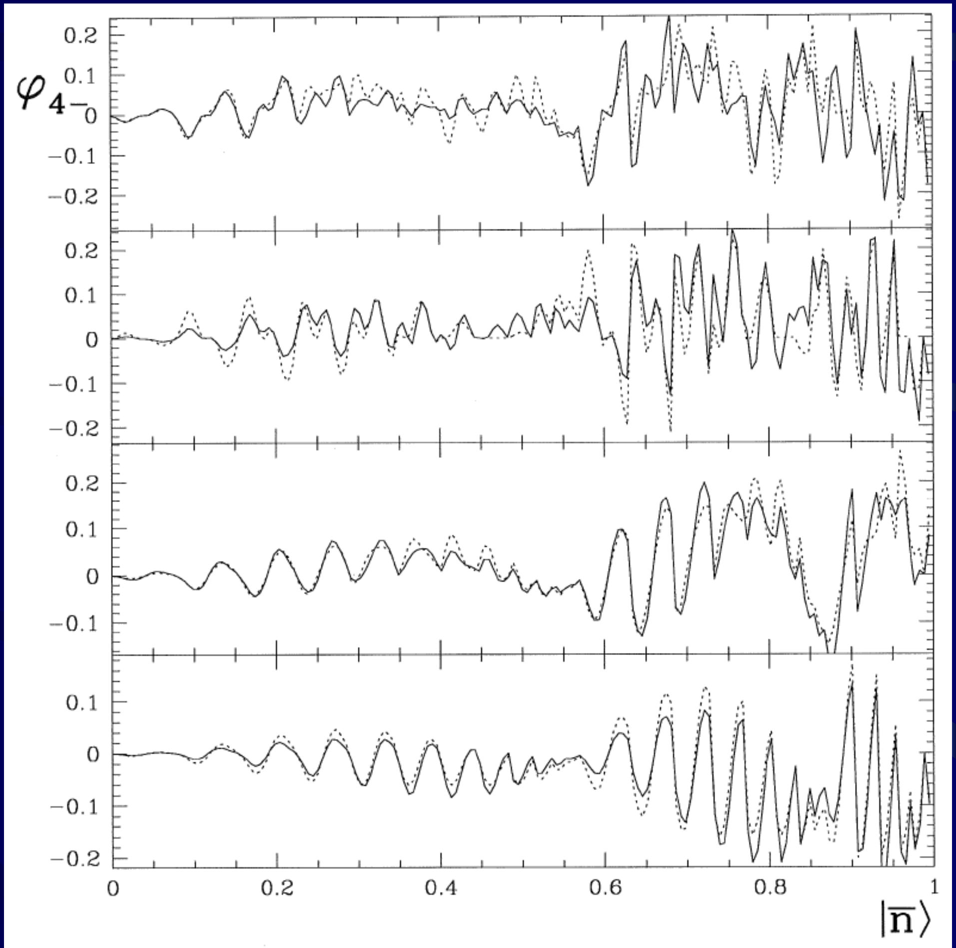


Massive theory

Works! Massive Four-Parton Eigenfunctions

Numerical (solid) vs. Algebraic (dashed)





T+ (even) under string reversal **(odd) T –**

The Formal Solution

To formalize our approach, it is convenient to make the inversion of frequencies explicit with the operator

$$\mathcal{I} : |n_1, n_2, \ldots, n_{r-1}\rangle \rightarrow |-n_1, -n_2, \ldots, -n_{r-1}\rangle. \tag{24}$$

Clearly, $\mathcal{I}$ is a Z_2 operator, and states even and odd under $\mathcal{I}$ simply represent cosine and sine wavefunctions, respectively. We can then write down an orthonormal set of basis states in all sectors of the theory, characterized by their Z_2 quantum numbers (T, I, S) under the symmetry transformations $\mathcal{T}$, $\mathcal{I}$, and $\mathcal{S}$, and their excitation numbers n_i

$$\left\{ \frac{(1 + T\mathcal{T})(1 + I\mathcal{I})}{\sqrt{2r!\mathcal{N}}} \sum_{k=0}^{r-1} (-)^{(r-1)k} \mathcal{C}^k \sum_{i}^{N(r)} S_i \mathcal{S}_i |r\rangle \right\}, \tag{25}$$

where $\mathcal{N}$ is the volume of the Hilbert space in the r parton sector,

$$\mathcal{N} = \int_0^{1/r} dx_1 \left(\prod_{i=2}^{r-1} \int_{x_1}^{1-(r-1)x_1 - \sum_{j=2}^{i-1} x_j} dx_i \right) = \frac{1}{r!}, \tag{26}$$

Start with asymptotic Theory:

H =H +H PC,s

n is

eory splits into

decoupled secto

$$\frac{M^2}{g^2 N}\phi_3(x_1,x_2,x_3) = -\int_0^1 \frac{dy}{(x_1-y)^2}\phi_3(y,x_1+x_2-y,x_3) - \int_0^1 \frac{dy}{(x_2-y)^2}\phi_3(y,x_2+x_3-y,x_1) - \int_0^1 \frac{dy}{(x_3-y)^2}\phi_3(y,x_3+x_1-y,x_2)$$

r =3

- Wavefunctions

like integral equ

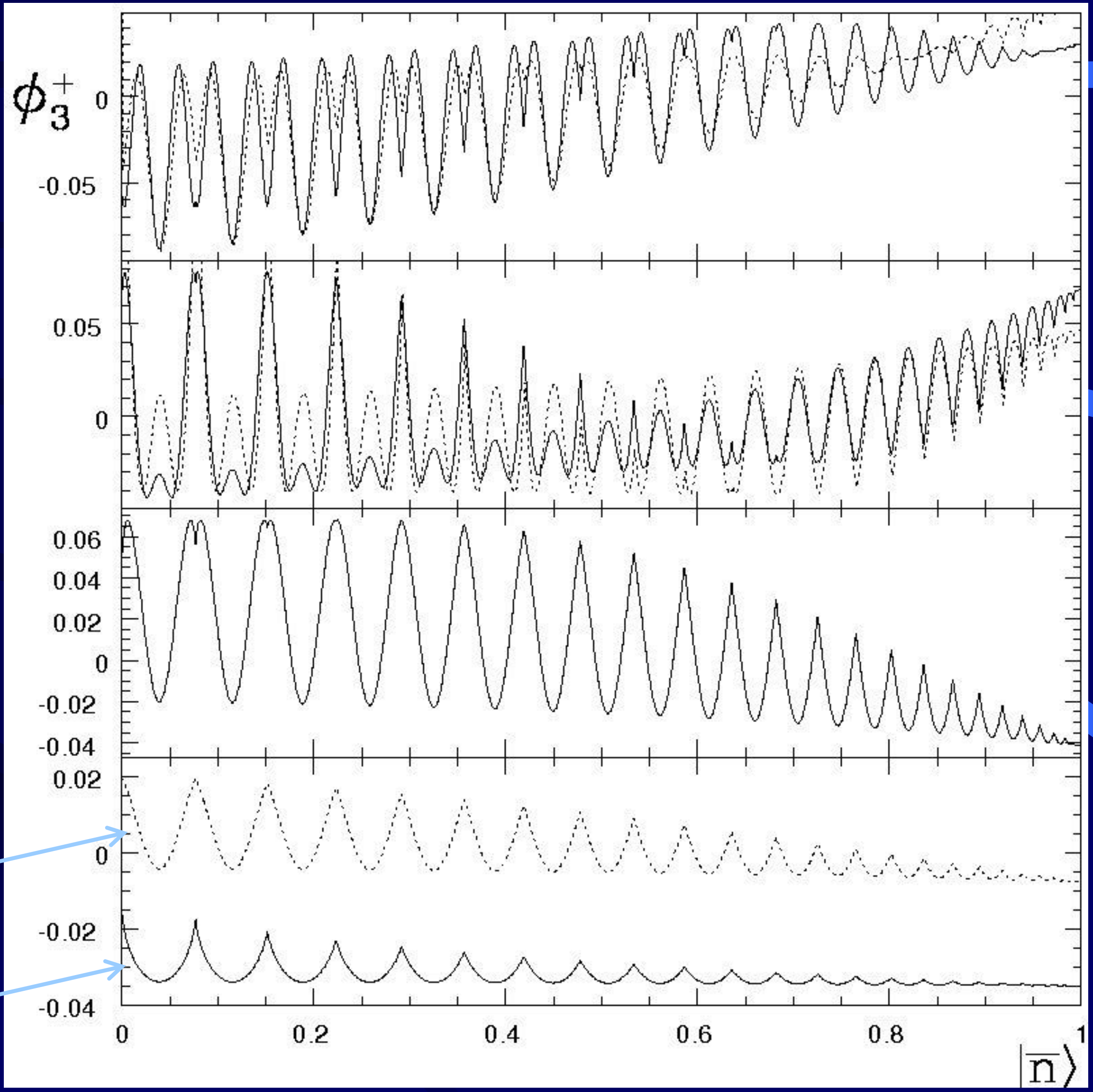
$$\frac{\pi M^2}{g^2 N_c}\phi_4 = \int_{-\infty}^{\infty} \frac{dy}{(x_1-y)^2}\phi_4(y,x_1+x_2-y,x_3,x_4) - \int_{-\infty}^{\infty} \frac{dy}{(x_2-y)^2}\phi_4(y,x_2+x_3-y,x_4,x_1) + \int_{-\infty}^{\infty} \frac{dy}{(x_3-y)^2}\phi_4(y,x_3+x_4-y,x_1,x_2) - \int_{-\infty}^{\infty} \frac{dy}{(x_4-y)^2}\phi_4(y,x_4+x_1-y,x_2,x_3).$$

r =4

$$M^2\phi_r(x_1,\ldots,x_r) = -\sum_{i=1}^{r}(-1)^{(r+1)(i+1)}\int_{-\infty}^{\infty}\frac{\phi_r(y,x_i+x_{i+1}-y,x_{i+2},\ldots,x_{i+r-1})}{(x_i-y)^2}dy,$$

Generalize: add non-singular operators

- Adding regular operators gives similar eigenfunctions but shifts masses dramatically



- Dashed lines: EFs with just singular terms (from previous slide)
- Here: shift by constant WF of previously massless state