Coffee Hour Problems

Edited by Matthew McMullen

Fall 2013

Week 1. *Proposed by Matthew McMullen.*

Find all pairs of positive integers x and y such that $20x + 17y = 2017$.

Week 2. *Proposed by Jeremy Moore.*

Find all pairs of positive integers a and b, with $a < b$, such that 4 is the largest integer that divides both a and b and 480 is the smallest positive integer that is divisible by both a and b. (In other words, $\gcd(a, b) = 4$ and $\mathrm{lcm}(a, b) = 480$.)

Week 3. *Proposed by Matthew McMullen.*

Find, with proof, the last two digits of 2^{2013}.

Week 4. *Proposed by Matthew McMullen.*

An online quiz consists of five multiple-choice questions, each with four possible choices. You can take the quiz three times (with different questions each time), and your best single score becomes your final grade for the quiz. You haven't studied at all, so you randomly guess the answers each of the three times you take the quiz. What is the probability that your final grade for the quiz is 20%?

Week 5. *Proposed by Matthew McMullen.*

If you look at a table of critical values of the Student's t-distribution, you will find that, with one degree of freedom, the value of t that has area 0.025 to its right is approximately 12.706. With two degrees of freedom, this value is approximately 4.303. What are the exact values of these critical numbers?

Week 6. *Proposed by Matthew McMullen.*

In order to save time on a group stats project, you and your partner collect data separately. You ask 30 randomly-chosen students what their GPAs are, and you obtain a sample mean of 2.85 and a sample standard deviation of 0.45. Your friend asks 20 randomly-chosen students what their GPAs are, and she obtains a sample mean of 3.05 and a sample standard deviation of 0.55. Based only on this information, find the sample standard deviation of the GPAs of all 50 students.

Week 7. *Proposed by Matthew McMullen.*

Suppose $f(x) = \frac{ax}{bx^2+c}$ has an absolute maximum value of 2013 at $x = 2$. Find the coordinates of all of the inflection points of f.

Week 8. *Proposed by Matthew McMullen.*

The line tangent to the curve $y^3 + 3y = x^2$ at the point $(2, 1)$ intersects the curve at another point. Find the coordinates of this point.

Week 9. *Proposed by Matthew McMullen.*

Find all pairs of rational numbers (x, y) such that $y^2 = x^3 - 432$. (*Hint:* If $u = \frac{36+y}{6x}$ and $v = \frac{36-y}{6x}$, what is $u^3 + v^3$?)

Week 10. *Proposed by Dave Stucki.*

Show that

$$\sqrt[3]{\sqrt{108}+10}-\sqrt[3]{\sqrt{108}-10}=2$$

and

$$\sqrt[3]{2+\sqrt{-121}}+\sqrt[3]{2-\sqrt{-121}}=4.$$

Week 11. *Proposed by Matthew McMullen.*

A drinking glass has a volume of 30 cubic inches. The top edge of the glass is a circle of radius 1.5 in., and the bottom of the glass is an ellipse with major radius 1.25 in. and minor radius 1.0 in. Assuming the top circle linearly tapers to the bottom ellipse, find the height of the glass.

Week 12. *Purdue University Problem of the Week.*

A standard six-sided die is rolled forever. Let T_k be the total of all the dots rolled in the first k rolls. Find the probability that one of the T_k's is eight.

Week 13. *Purdue University Problem of the Week.*

Let R be the region below the graph of $y = x$ and above the graph of $y = 3^x - x - 1$, for $0 \leq x \leq 1$. Find the volume of the solid obtained by rotating R around the line $y = x$.

Week 14. *Proposed by Matthew McMullen.*

You randomly throw a dart at an equilateral triangle-shaped dartboard. What is the probability that your dart is closer to the center of the dartboard than to any edge?

Week 15. *Proposed by Matthew McMullen.*

A rectangular box has dimensions $11.25''$ by $11.25''$ by $3''$. Is it possible to fit ten cubic blocks, each with side length $3''$, in the box? If so, explain how to do it; if not, explain why not.