Coffee Hour Problems of the Week

Edited by Matthew McMullen

Otterbein University

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Week 1. *Classic puzzles proposed by Matthew McMullen.*

(a) Three boxes of fruit are labeled *Apples*, *Oranges*, and *Apples and Oranges*. Each label is wrong. By selecting just one fruit from just one box, how can you determine the correct labeling of the boxes?

(b) There are two boxes: one marked A and one marked B. Each box contains either $1 million or a deadly snake that will kill you instantly. You must open one box. On box A there is a sign that reads: "At least one of these boxes contains $1 million." On box B there is a sign that reads: "A deadly snake that will kill you instantly is in box A." You are told that either both signs are true or both are false. Which box do you open?

Week 2. *Proposed by Matthew McMullen.*

Baseball player Adam Dunn is having a horrible year. According to an article in the August 28, 2011 *Columbus Dispatch*, "The Chicago White Sox slugger possessed a .165 average through Thursday, needing hits in 16 consecutive at-bats just to pass the Mendoza line of .200." How many hits and how many at-bats did Dunn have through Thursday?

Week 3. *Proposed by Matthew McMullen.*

Find

$$\lim_{x\to\infty}\left(e^{-2011}\left(1+\frac{2011}{x}\right)^{x}\right)^{x}.$$

Week 4. *Proposed by Matthew McMullen.*

Is it possible to find a strictly increasing, unbounded sequence of positive numbers, (x_n), such that

$$\sum_{n=1}^{\infty}\left(1-\frac{x_n}{x_{n+1}}\right)$$

converges?

Week 5. *Proposed by Matthew McMullen.*

Let $P(x)$ be a polynomial of degree 2011 satisfying $P(k) = k$, for all $k = 1, 2, \ldots, 2011$, and $P(0) = 2$. Find $P(-2)$.

Week 6. *Proposed by Jeremy Moore.*

Let $\mathbf{R}$ be the set of all real numbers. Then $M_n(\mathbf{R})$ denotes the set of all $n \times n$ matrices with real-number entries. Consider this set as a vector space over $\mathbf{R}$. Let $n = 2$, and suppose you have a collection of linearly independent, invertible matrices in this vector space. Are the inverses of these matrices linearly independent? What if $n > 2$?

Week 7. *Proposed by Devin Fraze.*

You have just solved a Sudoku, but your friend doesn't believe you. Since your friend would like to try the same puzzle later, how can you prove to her that you have solved it without giving away any specific information about the solution?

Week 8. *Proposed by Matthew McMullen.*

Imagine a game where six fair dice are rolled. If exactly four different numbers appear, you win; otherwise, you lose. What are your odds of winning this game?

Week 9. *Proposed by Matthew McMullen.*

Imagine an experiment where six fair dice are rolled. The random variable x represents the number of different numbers that appear (so $x = 1, \ldots, 6$). Find the probability distribution for x, the expected value of x, and the standard deviation of x.

Week 10. *Taken from Stewart's* Calculus.

Find the two points on the curve $y = x^4 - 2x^2 - x$ that have a common tangent line.

Week 11. *Proposed by Matthew McMullen.*

Suppose that z is a non-zero complex number. Classify all complex numbers w such that

$$\frac{1}{z+w} = \frac{1}{z} + \frac{1}{w}.$$

Week 12. *Proposed by Matthew McMullen.*

Find all real numbers a such that $(\arcsin \sqrt{a}\,)^2$ is real. (By $w = \arcsin z$, we mean the principle value of the complex extension of $y = \arcsin x$.)

Week 13. *Ancient Chinese problem.*

A band of 17 pirates stole a sack of gold coins. When they tried to divide the fortune into equal portions, 3 coins remained. In the ensuing brawl over who should get the extra coins, one pirate was killed. The wealth was redistributed, but this time an equal division left 10 coins. Again an argument developed in which another pirate was killed. But now the total fortune was evenly distributed among the survivors. What was the least number of coins that could have been stolen?

Week 14. *Proposed by Matthew McMullen.*

Determine all y such that

$$2011^{y/x} = \frac{2011^y}{x}$$

for exactly one x.

Week 15. *Proposed by Matthew McMullen.*

Find the number of ordered pairs of integers, (x, y), such that

$$2011 \leq \sqrt{x} + \sqrt{y} \leq 2012.$$