Coffee Hour Problems of the Week

Edited by Matthew McMullen

Otterbein College

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Week 1. *Proposed by Matthew McMullen*

You and a friend order a perfectly round 14-inch (in diameter) pizza cut into six congruent slices. The 14 inches includes a 1-inch-wide ring of outer crust. There is one piece left that you want to split, but you are tired of the crust and don't want any more of it. Describe all ways to divide the non-crust part of the slice into two equal-area pieces, one of which has no part of the crust on it, using a single straight-line cut.

Week 2. *Proposed by Matthew McMullen*

Find all four-digit numbers, N, such that the number formed by writing the digits of N backwards is a multiple of N.

Week 3. *2010 AIME II.*

Find the smallest positive integer n with the property that the polynomial $x^4 - nx + 63$ can be written as a product of two nonconstant polynomials with integer coefficients.

Week 4. *Proposed by Matthew McMullen.*

Let $f(x) = \sqrt{x}$ and $g(x) = x + a$, where $a \neq 0$. Suppose there exists an x_0 such that $f(x_0) = g(x_0)$ and $f(f(x_0)) = g(g(x_0))$. Find a.

Week 5. *Proposed by Matthew McMullen* (*inspired by* AMM).

You have five balls, numbered 1 to 5, that you will put into five urns, also numbered 1 to 5. First, ball 1 is put in a randomly selected urn. Then, ball 2 is put in urn 2 if it is empty, otherwise it is put in a randomly selected empty urn. Then, ball 3 is put in urn 3 if it is empty, otherwise it is put in a randomly selected empty urn. Then, ball 4 is put in urn 4 if it is empty, otherwise it is put in a randomly selected empty urn. Finally, ball 5 is put in the last empty urn. The random variable X represents the number of balls whose number matches the number of the urn it is put in. Find the expected value of X.

Week 6. *Proposed by Matthew McMullen.*

Find the maximum value of

$$\sum_{n=0}^{\infty}(-1)^{\lfloor n/2 \rfloor}x^n,$$

where $-1 < x < 1$.

Week 7. *Proposed by Matthew McMullen* (*inspired by* College Math. J.).

Prove that a differentiable function on the real line is a quadratic function if and only if the intersection of any two of its tangent lines lies midway horizontally between the points of tangency.

Week 8. *From the 2010 Harvard-MIT Mathematics Tournament.*

Calculate

$$\sum_{n=2}^{\infty}\sum_{k=2}^{\infty}\frac{1}{k^n \cdot k!}\,.$$

Week 9. *From the 1993 Putnam Exam.*

The horizontal line $y = c$ intersects the curve $y = 2x - 3x^3$ in the first quadrant, creating two regions: the first region is bounded by the y-axis, the line $y = c$ and the curve; the other lies under the curve and above the line $y = c$ between their two points of intersection. Find c so that the areas of these two regions are equal.

Week 10. *Proposed by Matthew McMullen.*

Suppose that $y \geq 3$ and

$$\ln\left(\frac{x}{y}\right) = \frac{\ln x}{\ln y}.$$

Find the minimum possible value of x.

Week 10$^+$. *Proposed by Matthew McMullen.*

Find

$$\int_0^1 \sqrt{\left\{\frac{1}{x}\right\}}\, dx,$$

where $\{u\} = u - \lfloor u \rfloor$ denotes the fractional part of u. (**Note:** I can't find a closed form expression for this. If *you* can, send me an email at mmcmullen@otterbein.edu.)