Coffee Hour Problems of the Week
Edited by Matthew McMullen

Otterbein College

Autumn 2009

Week 1. *Proposed by Matthew McMullen.*

To gain admittance to a clandestine math society, you need to determine the secret number. This number has two digits and is **uniquely** determined by the answers to the following yes/no questions.

1. Is the number divisible by 2?
2. Is the number divisible by 3?
3. Is the number divisible by 5?
4. Is the number divisible by 7?

(a) What are the answers to the last three questions?

(b) The answer to the first question is "no". What is the secret number?

Week 2. *Proposed by Adam Wolfe and Matthew McMullen.*

A circle of radius 1 is externally tangent to a circle of radius 3 at the point P. Let l denote a line tangent to both circles that does not pass through P. Find the area of the region bounded by l and the two circles.

Week 3. *Proposed by Matthew McMullen.*

Which is larger, $2009^{2010^{2009}}$ or $2010^{2009^{2010}}$?

Week 4. *Proposed by Matthew McMullen.*

(a) Let (a_n) be a sequence of real numbers such that the sequence $(2a_{n+1} - a_n)$ converges. Does (a_n) necessarily converge?

(b) Let (a_n) be a sequence of real numbers such that the sequence $(a_{n+1} - 2a_n)$ converges. Does (a_n) necessarily converge?

Week 5. *Proposed by Matthew McMullen.*

In solving $ax^2 + bx + c = 0$, for $a \neq 0$, some students will subtract c from both sides, factor an x out of the left-hand side, set each factor equal to $-c$, and then solve these two equations! Classify all such quadratic equations where this erroneous method yields *both* correct solutions.

Week 6. *Proposed by Matthew McMullen.*

The weatherman issues the following statement: "There is a 40% chance of rain on Saturday and a 50% chance of rain on Sunday. If it rains on Saturday, however, it will be twice as likely to rain on Sunday than if it doesn't rain on Saturday." What is the probability that it will rain at least one day this weekend?

Week 7. *1987 Putnam problem B-1.*

Find

$$\int_2^4 \frac{\sqrt{\ln(9-x)}}{\sqrt{\ln(9-x)} + \sqrt{\ln(x+3)}}\, dx.$$

Week 8. *Proposed by Matthew McMullen.*

Find a continuous function $f(x)$ such that

$$2009 = \int_0^1 f(x)\, dx = \int_0^1 x f(x)\, dx = \int_0^1 x^2 f(x)\, dx.$$

Week 9. *2009 Harvard-MIT Mathematics Tournament.*

Let $\mathcal{R}$ be the region in the plane bounded by the graphs of $y = x$ and $y = x^2$. Compute the volume of the region formed by revolving $\mathcal{R}$ around the line $y = x$.

Week 10. *Proposed by Matthew McMullen.*

The polynomial
$$f(x) = x^4 - 2x^3 - 2009x^2 + \alpha x + \frac{11}{13}$$
is symmetric about some vertical line. Find α.