Coffee Hour Problems of the Week

Matthew McMullen - Editor

Otterbein College

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Week 2. *Proposed by Matthew McMullen.*

James, Shawn, and Larsa enter their 10 meter by 10 meter classroom to take a topology test. Their professor, Dr. James, tells them to sit so that the minimum of the distances between any two of them is maximized; i.e., to spread out as much as possible. Explain where they should sit in the room to meet Dr. James' request, and find the minimum of the distances between any two of the students when they are in this configuration.

Week 3. *Proposed by Zengxiang Tong.*

Determine all polynomials $P(x)$ such that, for all x,

$$(x-1)P(x+1)-(x+2)P(x)=0.$$

Week 4. *Proposed by Matthew McMullen (inspired by a talk at the 2007 Ohio-MAA Fall Meeting).*

Toss a fair coin until you get two heads in a row. Let $P(n)$ denote the probability that you have tossed the coin n times ($n \geq 2$). Find a formula for $P(n)$, and justify your result.

Week 5. *Proposed by Zengxiang Tong.*

Suppose $x_1, x_2, \ldots, x_7$ are integers such that

$$\sum_{n=1}^{7} x_n^3 = 0.$$

Prove that 3 divides the product $x_1 x_2 \cdots x_7$.

Week 6. *Proposed by Shawn Winigman (from Nick's Mathematical Puzzles).*

Describe the largest semicircle that can be inscribed in the unit square, and find the area of this semicircle.

Week 7. *Proposed by Matthew McMullen.*

It is well known that $22/7$ is an approximation for π. Find the best rational approximation for π with numerator and denominator both less than 1000 (and positive). (*NB*: Computer or calculator assistance is encouraged.)

Week 8. *Proposed by Matthew McMullen.*

Let P be an arbitrary point inside the region bounded by parallelogram $ABCD$. Let a denote the length of line segment $\overline{AP}$, b the length of $\overline{BP}$, c the length of $\overline{CP}$, and d the length of $\overline{DP}$. Prove that $a^2 + c^2 = b^2 + d^2$ if and only if $ABCD$ is a rectangle.

Week 9. *Proposed by Dave Stucki.*

For both of the following sequences, find the missing term and describe the general pattern.

$$15, 23, 27, 29, 30, 39, 43, 45, 46, 51, 53, 54, ?, 58, 60, 71, 75, \ldots$$

$$31, 47, 55, 59, 61, 62, 79, 87, 91, 93, 94, 103, ?, 109, 110, 115, 117, \ldots$$