Coffee Hour Problems of the Week

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Week 1. *Proposed by Matthew McMullen.*

Can you write 2008 in the form $a^2 - b^2$, where a and b are positive integers? If so, find *all* such a and b; if not, explain why not.

Week 2. *From the 2008 Iowa Mathematics Competition.*

Write 2008 as a sum of consecutive positive integers.

Week 3. *Proposed by Tom James.* (*Quoted from* http://members.cox.net/fathauerart/FractalCrystalArt.html.)

A fractal "is constructed by starting with a [unit] cube and placing a half-scale cube on the center of each face. The second-generation cubes have the same orientation as the first-generation cube. Third-generation cubes again scaled by half are placed on each unoccupied face of a second-generation cube. This process is continued *ad infinitum* to form a 'fractal crystal'."

Find the volume of the fractal crystal.

Week 4. *Proposed by Matthew McMullen.*

Show that

$$\int_0^1 \Big[\ln(1/x)\Big]^n dx = n!$$

for every nonnegative integer n.

Week 5(a). *Proposed by Matthew McMullen.*

Find all real numbers x such that

$$\arccos x = \arctan x.$$

Week 5(b). *Proposed by Dave Deever.*

Prove that

$$\arctan 1 + \arctan 2 + \arctan 3 = \pi.$$

Week 6. *From the Iowa Mathematics Competition.*

Chicken McNuggets from McDonald's can be ordered in buckets of 6, 9, or 20. If you need 2008 McNuggets, for example, you could order 98 buckets of 20, 4 buckets of 9, and 2 buckets of 6. Find, with proof, the largest number of McNuggets you *couldn't* order.

Week 7. *Proposed by Matthew McMullen.*

A not-so-bright student was asked to solve the following equation (for x) on a MATH 115 exam:

$$\log x - \log 2008 = \log A - \log(x - 516).$$

To solve this equation, he canceled every "log" and solved the resulting equation for x. Amazingly, he got the correct answer! Find A.

Week 8. *Proposed by Matthew McMullen.*

The equation $\sin x = Ax$, where $A > 0$, has exactly nine solutions. Approximate A to five decimal places.

Week 9. *Proposed by Matthew McMullen.*

If $f(x)$ and $g(x)$ are differentiable at a and $f(a) = g(a)$, then the **angle between** f **and** g **at** $x = a$ is defined to be the acute, or possibly right, angle between their tangent lines at $x = a$.

Find, to the nearest tenth of a degree, the angle between $y = x^2$ and $y = \sqrt{x}$ at $x = 1$.

Week 10 and beyond. *Question by Matthew McMullen.*

Let S be the set of all functions, f, that are continuously differentiable and nonnegative on $[0, 1]$ and that satisfy $\int_0^1 [f(x)]^2 dx = 1$. Find, with proof,

$$\min_{f \in S} \int_0^1 f(x)\sqrt{1 + [f'(x)]^2}\, dx.$$