Coffee Hour Problems of the Week

(prepared by Matt McMullen)

Otterbein College

Autumn 2007

Week 2: Describe the nth term in the sequence

$$1, 2, 6, 12, 60, 60, 420, 840, 2520, 2520, \ldots$$

and find the next four terms.

(*Original problem proposed by Matt McMullen.*)

Week 3: The case of the missing square. See [**1**] for details.

(*Classical problem proposed by Dave Stucki.*)

Week 4: Prove that, for all positive integers n, and all valid x,

$$\frac{1}{\sin 2x} + \frac{1}{\sin 4x} + \cdots + \frac{1}{\sin 2^n x} = \cot x - \cot 2^n x.$$

(*1966 IMO problem proposed by Zengxiang Tong.*)

Week 5: Find

$$\sum_{n=1}^{2007} \frac{5^{2008}}{25^n + 5^{2008}}.$$

(*Original problem proposed by Zengxiang Tong.*)

Week 6: Show that if there are six people in a room, then there are three people that either mutually know each other or mutually are strangers to each other.

(*Classical problem proposed by Tom James.*)

Week 7: Describe the set of points inside a square of area one that are closer to the center of the square than to any edge of the square. (**Bonus:** Find the area of this set.)

(*Variation on a 1989 Putnam problem proposed by Matt McMullen.*)

Week 8: Fix $h, w > 0$ and the point (x_2, y_2). Let R denote the rectangle centered at (x_2, y_2) with width w and height h. Let (x_1, y_1) be any point outside R, and fix $l > 0$ and $0 < \theta < \pi/2$. Find coordinates p, q, and r, where p lies on R and on the line segment connecting (x_1, y_1) and (x_2, y_2); and p, q and r make up the head of the arrow pointing from (x_1, y_1) to p with fan-out θ ending l from the tip.

(*Original problem proposed by Duane Buck.*)

Week 9: Show that

$$\int_0^1 \frac{4x^3 \left(1 + x^{4(2006)}\right)}{(1+x^4)^{2008}}\, \mathrm{d}x = \frac{1}{2007}.$$

(*Problem from* Math Horizons *proposed by Zengxiang Tong.*)

References

[1] `http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibpuzzles2.html#jigsaw3`.