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**Volume Two , Number One**  
**March 2008**

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**Using Excel VBA Function**

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**by Larry George, Ph.D.**

**Reliability Symposium**

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Submit manuscript in MS Word, and MS Excel figures and charts are appropriate. If an equation or two is required use Microsoft Equation editor.. E-mail submittal to wmsent20@sbcglobal.net

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# Determine Exact Poisson C.I Using Excel VBA Function

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## Overview

This article examines the development of an Excel VBA function for computing the CI for a single Poisson rate. This article includes how to trap a known error in the Excel inverse chi-square function, and substitute an accurate inverse chi-square approximation. When the built-in Excel function has an error, this is caught by the VBA code so that the user never sees the error. Some Excel VBA update information is provided for the new Office 2007 which is a considerable change from the prior Microsoft Office interfaces.

The source of the data we introduce in our example computations is available in our companion article published in the March 2008 issue of *Reliability Review*

## Testing Goodness-of-Fit

When counting any type of event, occurrence, or failure a Poisson distribution is often used. The Poisson distribution is a flexible discrete distribution that may adequately model the failure events; however, this must be tested. A chi-square goodness of fit test is frequently used on a discrete distribution like the Poisson. This requires binning or collecting the number of observations that fall into various cells. A more powerful approach to assess the Poisson fit is to analyze the fit of the inter-event exposure variable to the continuous exponential distribution. It is easier to assess if the continuous distribution adequately models the inter-event data. No lumping of data into bins or cells is required. This is a more powerful statistical test and one that removes any subjectivity over how to create the bin widths needed for the chi-square goodness of fit test (D'Agostina and Stephens, 1986; Huber and Glen, 2007; Stephens, 1974). Such tests are necessary for each exposure variables of interest. Examples of goodness of fit tests are found in Eschenbach and Harper (2006) and to a lesser extent in Harper and Eschenbach (2007). Our personal experience has been that often the Poisson is found to be reasonable via a goodness of fit test (chi-square on the discrete Poisson or the Anderson-Darling or Kolomorgorov-Smirnoff tests on a continuous exponential). However there are instances where this is not the case. There seems to be a dearth of good discrete distributions when one compares the common discrete distributions to the plethora of continuous distributions. A more flexible option is a compound or mixed Poisson distribution (ch 5, Clark and Harper, 2000; or section 8.2.5, Johnson, Kemp, and Kotz, 2005). This can provide a longer tail than the standard Poisson to model rarer large values not uncommon in some disciplines such as with geologic data.

## Exact Poisson CI Useing Excel VBA (Cont'd)

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### Exact Poisson Confidence Intervals

Once a Poisson distribution has been justified, one can begin to estimate the occurrence rate. The rate is the mean of the Poisson distribution which is the ratio of the number of occurrences over the summed exposure variable. Care must be given to determine if a time varying Poisson mean (also known as a non-stationary Poisson process or a non-homogeneous Poisson process) is needed. If the assumption of a stationary non-changing mean is not appropriate then intervals of quasi-stationarity might be found for which a given Poisson rate is reasonable

Building from Johnson, Kemp and Kotz (2005, pp. 176) and Buchan (2004) the formulas below explicitly address the incorporation of an exposure variable. The first formula for  $\lambda_L$  represents an exact lower confidence bound for the mean Poisson rate  $\lambda$  while the second for  $\lambda_U$  is the formulation for the upper confidence bound. Taken together these form the 100(1- $\alpha$ )% confidence interval. Dividing  $\alpha$  equally into each tail results in a two-tailed exact confidence interval for the Poisson rate. These confidence intervals are based on the chi-square ( $\chi^2$ ) distribution.

$$I_L = \left( \frac{\frac{1}{2} \chi^2_{2x, \frac{\alpha}{2}}}{\sum \text{Exposure Variable}} \right); \quad I_U = \left( \frac{\frac{1}{2} \chi^2_{2(x+1), 1-\frac{\alpha}{2}}}{\sum \text{Exposure Variable}} \right)$$

Generally in statistics increasing the sample size decreases the width of confidence intervals. In these equations, the subscript x is the number of events (such as oil spills or failures), and is the basis for the number of degrees of freedom for the chi-square distribution. To check that this formula behaves as expected, assume that the number of occurrences and the amount of exposure are both doubled (which keeps the rate constant). Doubling the number of degrees of freedom more than doubles the lower  $\chi^2$  value. Since the exposure also doubled, the lower limit goes up and is closer to the estimated average. In like fashion doubling the number of degrees of freedom less than doubles the upper value, so the upper limit has decreased.

The chi-square value needed above is what is called the inverse chi-square. By this it is meant that the user will provide the appropriate confidence level desired (which in turns gives the value needed) and the number of incidents (x above). Then the inverse routine provides the corresponding value of the chi-square distribution.

### Excel VBA Solution Addressing Problems with Excel's Inverse Chi-Square Function

The formula for the exact Poisson confidence interval has two major components. The first is the inverse chi-square distribution that is addressed in this article. The second is the sum of the relevant exposure variable (often total time on test) which can be more difficult in practice than one might initially anticipate. While this is not the major focus of this article, its importance and potential challenges should not be overlooked.

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The Poisson confidence limits in the prior section would not be hard to implement in Excel if Excel's chi-square inverse routine was error free. For large degrees of freedom, Excel's inverse chi-square distribution (even in Excel 2007) aborts and sends an error message to the user. Iain Buchan (2004) offered a VBA solution but it required having a very specific U.K. statistical software package StatsDirect that could be linked to and called by VBA. In our approach detailed below no other statistical package is required.

The approach taken was to modularize the VBA code keeping each subprogram as simple as possible. This aids the debugging as it permits isolation of any errors. The following general steps were used in the development of the VBA functions available free at <http://faculty.otterbein.edu/WHarper/>. While the last step below is purely cosmetic, it has been found to save consider time when pulling together reports. The user can bypass this last step if desired. Once installed this VBA code performs just like any Excel function.

Check user inputs for possible errors and report them.

Attempt to use the Excel CHIINV

If an error occurs in the Excel CHIINV

Trap the error so that the user does not see the error message

Use the approximation given below for the inverse chi-square

Compute the lower and upper confidence interval values.

Concatenate the results to provide a nice looking two-tailed 100(1- $\alpha$ )% confidence interval.

Checking function input ensures that the user does not get thrown in the midst of the Visual Basic Editor if an error occurs. It is the responsibility of the code developer to provide meaningful error messages as seen in an example later. The Excel CHIINV function aborts for large degrees of freedom. Buchan (2004) for example reports failures for CHIINV(0.975, 932). When this occurs, the user must be protected. The VBA code uses an On Error check to capture such events and then uses the accurate chi-square approximation given below. The user may call separate VBA functions for the lower and upper confidence limits of interest or use a function for step 5 above providing a cosmetically appealing two tailed confidence interval in a single Excel cell fashioned to meet the user's desired number of decimal digits.

The approximation used to the inverse chi-square is found in Johnson, Kotz, and Balakrishnan (1994). It is the Wilson-Hilferty (1931) approximation and follows:

$$F_{\chi^2_n}(x) = \Phi \left( \sqrt{\frac{9n}{2}} \left\langle \left( \frac{x}{n} \right)^{\frac{1}{3}} - 1 + \frac{2}{9n} \right\rangle \right) \text{ where } \Phi(x) \text{ is the standard normal distribution.}$$

From this the inverse chi-square distribution is approximated as shown below.

Continued on next page

## Exact Poisson CI Useing Excel VBA (Cont'd)

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$$X_{n,e}^2 = n \left( \sqrt{\frac{2}{9n}} \Phi^{-1}(e) + 1 + \frac{2}{9n} \right)^3$$

where  $\Phi^{-1}(e)$  is the lower e percentage point of the standard normal distribution.

Thus  $\Phi^{-1}(e)$  is the inverse of the standard normal distribution.

The following five functions and one subprogram comprise the downloadable VBA code that can be added to one's current Excel repertoire. There are multiple ways to do this such as storing these in the personal.xls (personal.xlsm in Excel 2007) or creating an Add-In.

Function PoissonCI\_Lower(NumIncidents, SumExposureVariable, ConfLevel)

Function PoissonCI\_Upper(NumIncidents, SumExposureVariable, ConfLevel)

Function PoissonCIText(NumIncidents, SumExposureVariable, ConfLevel, NumDigits)

Function ChiFix(prob, df)

Function ChiInvApprox(prob, df)

Sub CheckPoissonConfIntInput(NumIncidents, SumExposureVariable, ConfLevel)

The first two functions may be all that one directly calls from within Excel. The user supplied inputs are the number of events or incidents, the sum of the exposure variable, and the desired confidence level. These lower and upper Poisson confidence intervals place  $\alpha/2$  into the relevant tail and are thus based on the assumption that most users will want two-tailed confidence intervals. If this is not the case, the bold user can easily modify the VBA code or more simply change the confidence level provided. For example if one wanted an upper one-tailed 95% confidence interval, specification of 0.90 as the input confidence level accomplishes this automatically by putting  $(1 - 0.90)/2 = 0.05$  into what the function thinks is  $\alpha/2$  but instead is the user desired  $\alpha = 0.05$  for the one-sided interval.

The function PoissonCIText puts the full two-tailed confidence interval into a single Excel cell. This is accomplished by concatenation of the results of both the lower and upper confidence interval values along with parentheses and addressing the user specified number of decimal digits. The functions ChiFix and ChiInvApprox are called by the above functions. They could have been made sub procedures rather than functions, but there are times where one may want to directly find the inverse chi-square value. In VBA sub procedures operate as commands that may change many things such as the formatting of multiple cells or insertion of new worksheets. Functions however are passive in the sense that they return usually a single value in response to user input without changing formatting or the other physical aspects of the workbook. In ChiFix the following code is used:

## Exact Poisson CI Using Excel VBA (Cont'd)

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```
On Error GoTo UseApprox
```

```
ChiFix = WorksheetFunction.ChiInv(prob, df)
```

```
Exit Function
```

```
UseApprox:
```

```
ChiFix = ChiInvApprox(prob, df)
```

```
End Function
```

The On Error condition occurs when the Excel function ChiInv aborts. It then sends the program to the label UseApprox if an error occurs in this function. If the built-in Excel function ChiInv (which is also an approximation to the inverse chi-square) has an error (which it does for large degrees of freedom), the ChiInvApprox function is used. If ChiInv does not generate an error, its returned value is used and the function is left via the Exit Function statement.

A VBA subprogram CheckPoissonConfIntInput verifies user input and provides hopefully meaningful error messages to the user so that the user can figure out what to do. Also this keeps the user from entering the possibly intimidating domain of the Visual Basic Editor.

Figures 1, 2, and 3 are screen dumps illustrating both the types of visual interface the user may encounter using these functions. In the examples below the Insert Function approach was used instead of directly keying in the function name (which will also work fine). Note the helpful description of the PoissonCIText function which the developer should provide (Walkenbach, 2007; Harper & Eschenbach, 2007)

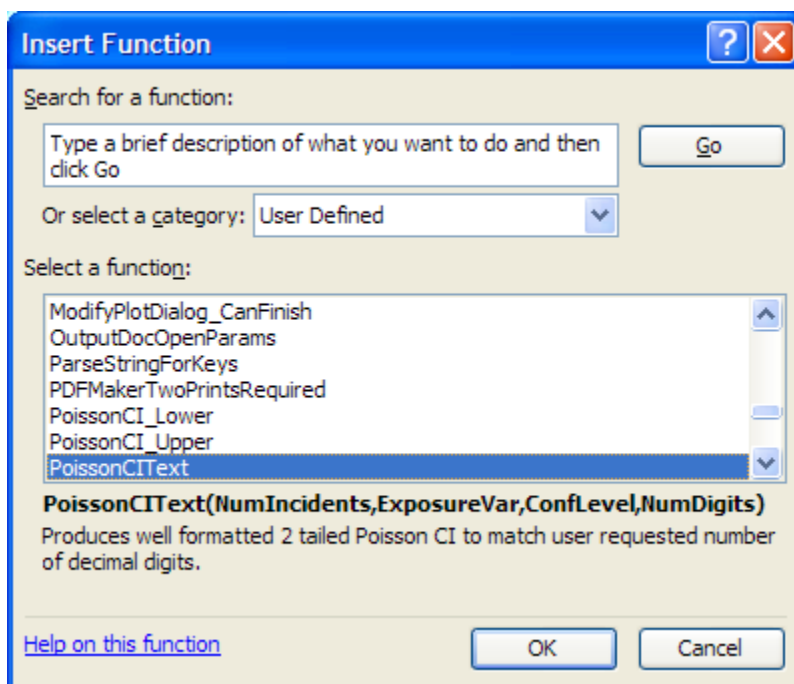


Figure 1. Insert Function use of Exact Poisson CI function.



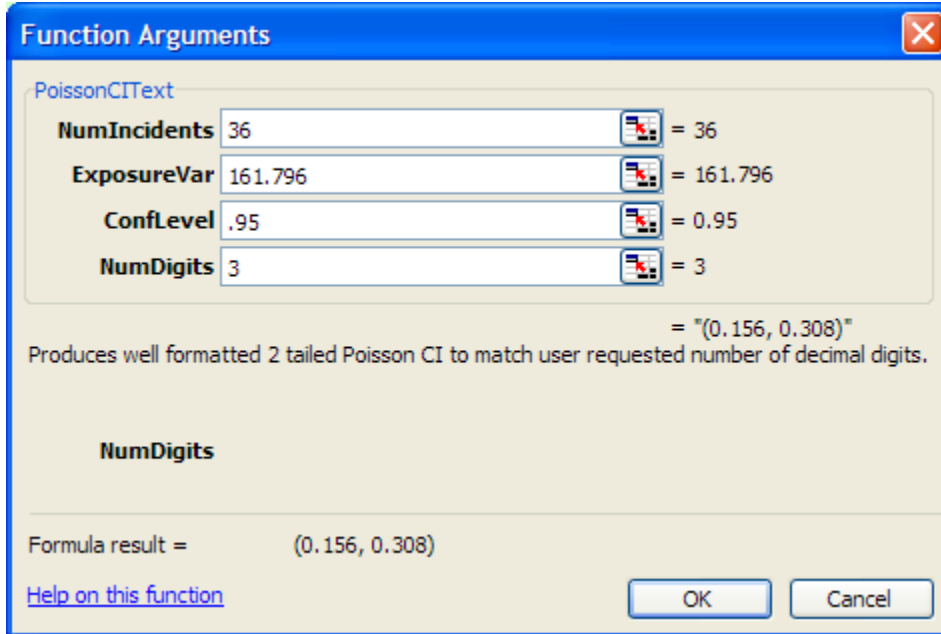
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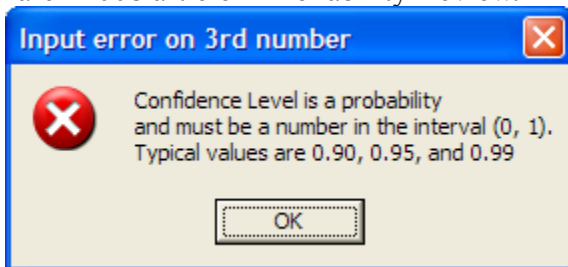
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**Figure 2. Exact Confidence Interval example for Oil Spill data**

Figure 2 shows the concatenated text output that will appear in the desired Excel cell of (0.156, 0.308) with the user specified number of decimal points. This is a handy way to quickly get nicely formatted output from Excel that can easily be dropped directly into a report. The data input into Figure 2 were taken from the Pipeline Spills data presented in Table 1 of our article entitled “Exposure Variables for Reliability Assessment: Exact C.I.” published in the March 2008 issue of Reliability Review. The results obtained in Figure 2 agree with those presented in the article. In this same manner the Confidence Intervals were computed for the remaining tables presented in our March 2008 article in Reliability Review.



**Figure 3. Sample Error Message.**

### Brief Comments about Office 2007

Office 2007 is working its way onto the computers of the world and has a different interface than what users have grown accustomed to. It is worth the transition if you need more rows or columns than available in Excel 2003. Excel 2007 has over 1,000,000 rows per worksheet and the number of columns has jumped from 255 (Column IV) in Excel 2003 to over 16,000 (Column XFD).



# Exact Poisson CI Useing Excel VBA (Cont'd)

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These Excel 2003 VBA routines will work with Excel 2007 but will lose the helpful text such as “Produces well formatted 2 tailed Poisson CI to match user requested number of decimal digits” seen in the Insert Function dialogue box above. Such comments can be easily reloaded manually; however, we provide both 2003 and 2007 versions on the web.

We are not in a position to offer detailed advice in this article on Excel 2007 VBA (see Walkenbach 2007) but expect learning time delays in the migration to Office 2007. Excel 2007 has more security based issues such the familiar .xls suffix being replaced with either .xlsx or .xlsm where .xlsm implies macro (VBA) code is included. VBA has changed little from 2003 and once in the Visual Basic Editor, the interface is the same as before. Have patience and share Office 2007 knowledge with others.

## Conclusion

Failure, event, or occurrence counts are important to product and system integrity. The development of rates and confidence intervals for appropriate exposure variables aids engineers and managers to predict reliability and protect our lives and the environment. Whenever possible, the quantification of uncertainty provided by sound CIs should be standard engineering practice.

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# Estimate Part Reliability from Age at Replacement Databases

by Larry George, Ph.D.

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The objective is to estimate the part reliability from hidden renewal process data by nonparametric maximum likelihood. Age-at-renewal databases do not offer renewal counts, only product ages-at-part renewals and part name or number. The same part may be used in several locations, but the data does not tell in which location the renewal occurred. This article applies to any product which experiences part renewals, including products having the same parts possibly used in different locations. An additional information constraint is that the only usage data is product ages at part renewals and part name or number. An equivalent term for part renewal would be part replacement; that is removal of the failed part and replacement with a new part.

The same situation arises in biostatistics. A breast cancer database records ages at occurrences by patient names but doesn't specify which breast. Does the recurrence occur in the same breast or the other? The distribution of ages at occurrences helps distinguish recurrence in the same breast from new occurrence. Estimating "reliability" for breast cancer helps determine whether lumpectomy is adequate.

## Background

Statisticians estimate reliability from random samples of ages at renewals using "life tables." Reliability statisticians estimate reliability from randomly censored random samples using the Kaplan-Meier nonparametric maximum likelihood estimator (npmle). If data come from renewal processes, they use parts' ages between successive renewals [Pena et al.]. I estimate reliability from product ships and part renewal counts: nonparametric least squares estimator (nplse) and npmle, [George 1983 and 2002].

Often databases do not specify part ages at renewals, only ages at renewals of "tools." However, the semiconductor capital equipment industry calls products, "tools." Some tools are fabulously expensive tools! They are referred to as fabs. (ent. "fabricated). The databases tell part ages at renewals, probabilistically. Estimators from ships and returns counts are not using all the information in that data. Figure 1 shows the hierarchy of information in different data.

Part ages at renewals	Censored part ages at renewals and survivors' ages	Tool ships and tool ages at part renewals	Part ships and renewal counts
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More

Information

Less

**Figure 1. Relative information in alternative kinds of data**

Reliability engineers need nonparametric estimates of part reliability functions, especially those that fail often. Non-parametric estimators avoid unwarranted assumptions and preserve all information in data.

Databases collect only data required for generally accepted accounting principles, not for statisticians or reliability engineers. Management buys the database regardless of whether it provides the data we

# Estimate Part Reliability from Age at Replacement Databases (Cont'd.)

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need. We must cope!

## Simple Example

Table 1 shows typical data. Four tools were shipped. Each uses two of a part. Each part generates a renewal process. Column A is the calendar quarter when tools were shipped. Column B counts how many parts were placed in service. The (truncated) triangular matrix counts part renewals from tools shipped each calendar quarter. This data gives the ages at which tools failed due to the part failures but not all part ages at renewals.

Table 1. Typical ships and return for each calendar quarter

Quarter	Ships					q1-06	q2-06	q3-06	q4-06	q1-07	q2-07	q3-07	q4-07
q1-05	2	0	0	0	0	0	0	0	0	0	1	0	0
q2-05	0		0	0	0	0	0	0	0	0	0	0	0
q3-05	0			0	0	0	0	0	0	0	0	0	0
q4-05	2				0	0	1	0	0	0	0	0	0
q1-06	2					0	0	0	0	0	1	1	0
q2-06	2						0	0	0	0	1	0	0

We don't know whether the two renewals of the tool shipped in q1-06 were in the same or different locations. Figure 2 shows the alternatives:

Part renewals were in different locations at ages 6 and 7 quarters. Survivors' ages were 2 and 1 respectively.

Both part renewals were in the same part location at ages 6 and 1 quarters, and the part in the other location didn't fail. Survivors' ages are 1 and 8 respectively.

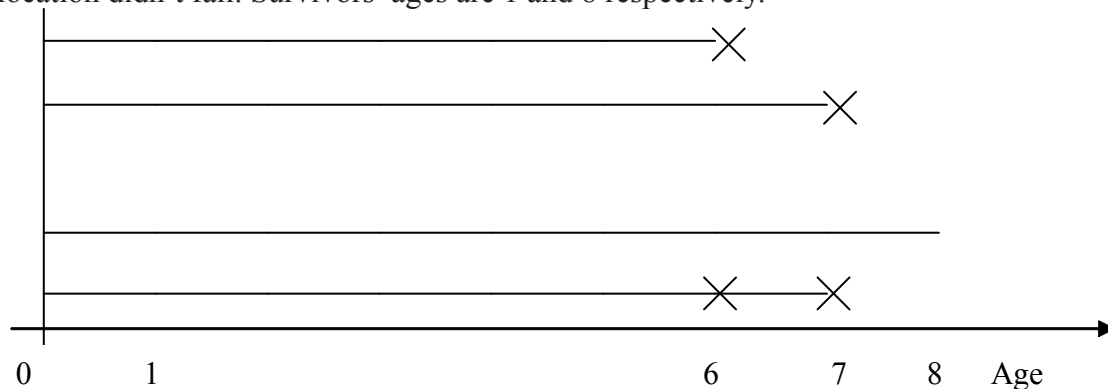


Figure 2. Alternatives for the q1-06 tool part ages at renewals. X denotes renewal.

# Estimate Part Reliability from Age at Replacement Databases (Cont'd.)

by Larry George, Ph.D.

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## EM Algorithm

The EM algorithm makes parameter estimates with missing data [Dempster et al.]. EM stands for “expectation-maximization”, of the likelihood function. The EM algorithm iterates expected value of alternative likelihood functions and maximizes over unknown parameters. In nonparametric statistics, the unknown parameters are the probability distribution function values of the part age at renewal. Maximum likelihood reliability estimation places all probability at observed ages at renewals.

Because table 1 data are ambiguous, there are two likelihood functions, with unequal probabilities. Enumerate the alternatives and compute their likelihoods, under all possible assumptions for the missing data. The likelihood functions are of the form  $L = \prod f_j \prod R_j$  where  $f_j$  represents the unknown probability distribution function values at observed renewal times, and  $R_j$  is the reliability function of the survivors' ages. The first product is over all renewals, and the second is over all survivors. This is the likelihood function of the censored-data Kaplan-Meier  $\text{npml}$ . The alternative likelihood functions differ, conditional on the hidden but assumed ages at renewals (figure 2).

The alternative probabilities of the q1-06 tool are

$$p1 = P[N1(8) = 1]P[N2(8) = 1] \text{ and } p2 = P[N1(8) = 2]P[N2(8) = 0]$$

where  $N(t)$  denotes the number of renewals for part locations. (Assume part locations generate independent renewals.) Probabilities from other tools are the same for both alternatives.

Renewal theory computes the probability distribution function of the renewal counting function  $N(t)$ ,  $P[N(t) = n] = P[\text{nth renewal is at age } \leq t \text{ and } n+1\text{st} > t]$ , in terms of convolutions of the cumulative distribution function,  $F(t) = \bar{1}R(t)$ .  $F_n^*(t)$  denotes the  $n$ th convolution. The formula is  $P[N(t) = n] = F_n^*(t)\bar{F}_{n+1}^*(t)$ .

The expectation step computes the expected (log) likelihood

$$E[\ln L] = p1 \ln L1 + p2 \ln L2$$

where  $\ln L1$  and  $\ln L2$  are the log likelihoods.

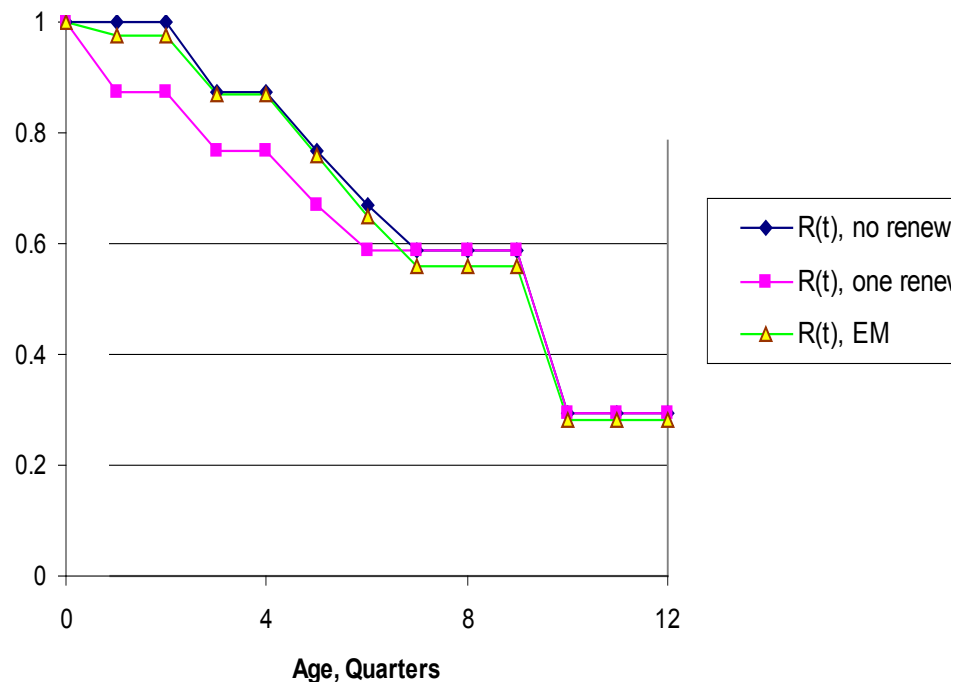
The maximization step finds the unknown parameter values that maximize  $E[\ln L]$  by varying the unknown probabilities in the formulas for  $L1$  and  $L2$ .

Start with the average of the alternative KM estimators. Iterate until convergence. There is no guarantee of convergence. To my surprise, the expected likelihood is not monotonically increasing, but the EM algorithm converged to a reasonable estimate.

Figure 3 shows the KM estimators for the two alternatives and the EM estimator, which initially lies between the two alternatives then below, always closer to the first alternative. This first alternative has almost twice the probability of the second alternative.

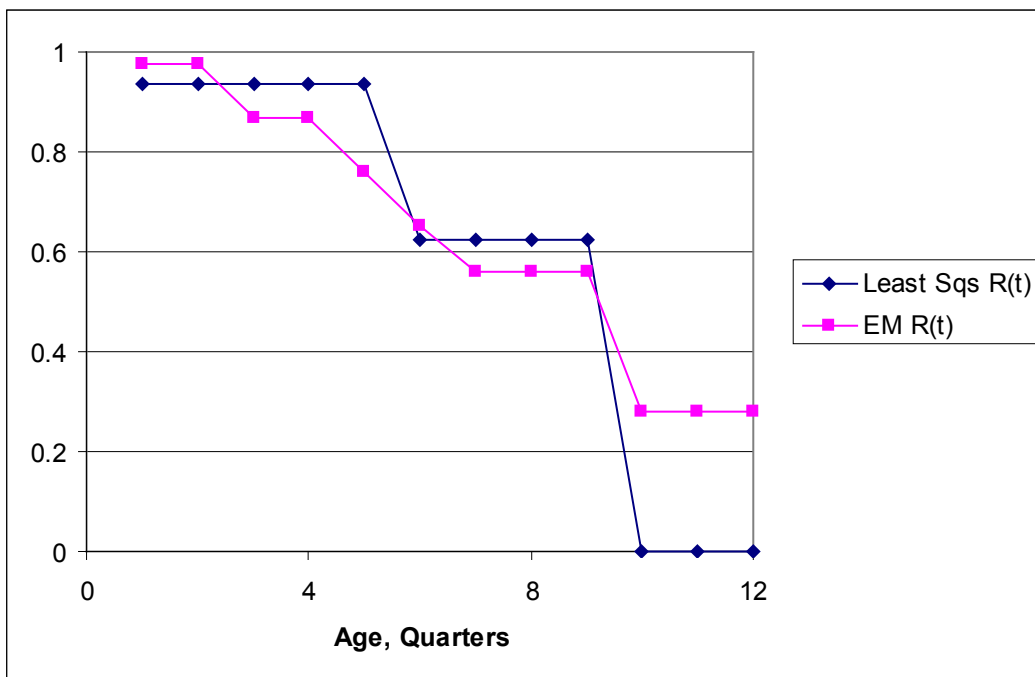
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by Larry George, Ph.D.



**Figure 3. Alternative nonparametric maximum likelihood reliability estimates**

Figure 4 shows the EM estimator and the nplse from ships and returns counts. The latter does not put probability at every age at renewal, unlike maximum likelihood estimators. The nplse is less precise; it does not use all information in the data.



**Figure 4. Compare EM npml and nplse from ships and returns counts**

# Estimate Part Reliability from Age at Replacement Databases (Cont'd.)

by Larry George, Ph.D.

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## Recommendations

Don't beg database designers or service people to include hidden data like part serial number or part location. Vendors such as Oracle, Agile, Sage CRM SalesLogix, and the like probably designed the database. They are not accustomed to responding to reliability engineers. Send data, and I will send back the EM npmle, free of charge. Please K.I.S.S. This could be a combinatoric nightmare.

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Larry is an ASQ certified reliability statistician doing business Problem Solving Tools. He has a Ph.D. in Industrial Engineering and Operations Research from UC Berkeley. He taught for 11 years, worked for a national laboratory for 11 years, and has been working in the real world for more than 20 years. The ASQ Reliability Division honored him in 1996 with the Allen Chop award and the ASQ elected him a Fellow in 2000. Larry crusades for applications of field reliability using available population data. Find more information at <http://www.fieldreliability.com>.

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# INTERNATIONAL CONFERENCE ON PROGNOSTICS AND HEALTH MANAGEMENT 2008

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## Important Dates

### **Draft Paper Due**

25 April 2008

### **Reviewer Feedback**

27 June 2008

### **Final Paper Due**

15 August 2008

### **Conference**

6 - 9 October 2008

### **PHM Challenge**

17 March - 2 June 2008

## Co-Located Events

NASA Aviation Safety -  
IVHM Project Annual Meeting

## **Call for Papers** **PHM08**

Marriott Tech Center, Denver, Colorado, USA

October 6-9, 2008

<http://www.phmconf.org>



IEEE Reliability Society is proud to sponsor the first annual International Conference on Prognostics and Health Management. PHM08 is intended for researchers, R&D engineers, and managers working in this emerging interdisciplinary field. The primary objectives of the conference are to:

- Deliberate and establish the scientific methodologies for PHM research;
- Foster collaboration and communication between academic, government, and industry PHM communities across the globe; and
- Identify innovative business approaches that utilize PHM methods and findings.

PHM08 will create a sociable and professional environment for the participants to connect with researchers in the field, forge new relationships, and deepen existing ones. The conference will cover a broad range of research and application topics:

### **Principles**

- Physics of failure
- Software failures
- Sensors
- Structural sensing
- Health management system design and engineering
- Modeling and simulation

### **Methods**

- Data-driven methods for anomaly detection, diagnosis, and prognosis
- Model-based methods for fault detection, diagnostics, and prognosis
- Standards and methodologies
- Automated reconfiguration
- Verification, validation, and maturation
- Component-level PHM
- Software health management
- PHM for electronics
- Structural health management

### **Results**

- Innovative applications
- Industrial applications
- Informed logistics
- Lessons Learned from PHM systems design and integration
- Systems and platform applications
- Component-level prognostic results

The conference will feature keynote presentations by senior leaders in the field, panel discussions, and a full day of tutorials free to all registrants. Leading companies and research institutions will exhibit their products and demonstrate their technologies during the event. A unique feature of the conference is the PHM Challenge, featuring cash prizes and open to everyone in the field. Winners of the PHM Challenge will be invited to present their methods and results in a special session.

Prospective authors are invited to contribute full length, original, high-quality papers through the conference web site. Submitted papers will be refereed by experts in the field based on the criteria of originality, significance, quality, and clarity. The conference proceedings will be published on CD for conference attendees, and in IEEE Xplore®.