Physics 171 **PRINCIPLES OF PHYSICS I** **AQ 2009**

Second Midterm Exam

November 9, 2009

Name Key

INSTRUCTIONS: Complete all problems. Show all work in the space provided. No books or notes allowed. Partial credit will be given for problems correctly set up even if incorrectly answered. No credit will be given for a numerical answer without supporting work. One point will be deducted from each answer without correct units. The time limit for this examination is 1 hour and 20 minutes.

Good luck!

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1. **Vector Multiplication**

Consider two vectors, **A**= 4.1 **i** - 1.7 **j** , and **B**, which has a length of 6.5 units and points in the negative x direction.

a Calculate the scalar product of **A** and **B**. (2 points)

$$B_x = -6.5 \qquad B_y = 0 \quad \Rightarrow \vec{A}\cdot\vec{B} = (4.1)(-6.5) + (-1.7)(0)$$

$$= \underline{\underline{-26.65}}$$

b. Find a vector **C** that is perpendicular to **A** and has a length of 2.5 units. (2 points)

$$|\vec{C}| = 2.5 = \sqrt{C_x^2 + C_y^2}$$

$$\vec{A}\cdot\vec{C} = 0 = A_x C_x + A_y C_y \Rightarrow C_y = -\frac{A_x}{A_y} C_x$$

close + for definitions

$$\Rightarrow C_x^2\left(1 + \frac{A_x^2}{A_y^2}\right) = (2.5)^2 \Rightarrow C_x = \frac{2.5}{\pm\sqrt{1+\frac{A_x^2}{A_y^2}}} = \underline{\underline{\pm 0.958}}$$

$$C_y = -\frac{4.1}{(-1.7)}\, 0.958 = \underline{\underline{2.309}}$$

2. **Gravitation**

The Moon (mass 7.35×10^{22} kg, radius 1740 km) takes 27.425 days to go once around the Earth (mass 5.99×10^{24} kg) while the latter circles the sun. In its orbit it is at a distance of 384,000 km from Earth. The gravitational constant is $G=6.67\times10^{-11}$ Nm^2/kg^2.

a. Calculate the potential energy of the Moon in the gravitational field of the Earth.
(2 points)

$$U_{grav} = -\frac{GmM}{r} = -\frac{(7.35\cdot10^{22}\,kg)(5.99\cdot10^{24}\,kg)}{3.84\cdot10^{8}\,m}\left(6.67\cdot10^{-11}\,\frac{Nm^2}{kg^2}\right)$$

$$= -7.65\cdot10^{28}\,Nm$$

$$= \underline{\underline{-7.65\cdot10^{28}\,J}}$$

b. Calculate the centripetal force that the Earth exerts on the Moon to keep it in orbit.
(2 points)

$$F = ma = m\frac{v^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{m\,4\pi^2 r}{T^2}$$

$$= \underline{\underline{1.99\cdot10^{20}\,N}}$$

$$T = (27.425\,d)\left(24\,\frac{h}{d}\right)\left(3600\,\frac{s}{h}\right)$$

$$= 2369520\,s$$

c. What is the acceleration due to gravity on the Moon? (*Hint: This is a property of the Moon, so it should only depend on its mass and radius – and G.*) (2 points)

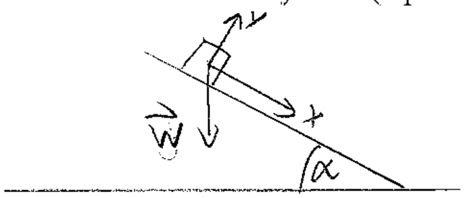
$m\,g_{moon}$ is the force of gravity on an object near the surface of the moon, so it has to be equal to $F_{grav} = G\frac{m\,M_{moon}}{r_{Moon}^2}$

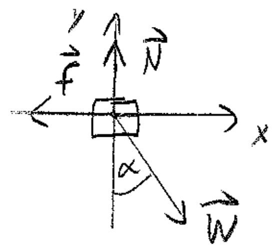
$$\Rightarrow \quad g_{moon} = G\frac{M_{moon}}{r_{moon}^2} = \underline{\underline{1.619\,\frac{m}{s^2}}} = 0.165\,g \approx \frac{1}{6}\,g$$

3. **Box on incline with friction**

A box of mass 2.6 kg, initially at rest, slides down an incline with an angle of 35° with an effective coefficient of kinetic friction of μ=0.3.
(Hint: Check your results by considering the case where the inclination is zero, i.e. the case of a box on a horizontal table)

a. Draw a free body diagram indicating all the forces acting on the box and indicate your choice of a coordinate system. (1 point)





b. What is the net force (magnitude and direction) that causes the box to slide down the incline? (3 points)

Direction is clearly positive x-direction.

$$\sum(\vec{F})_y = 0 = (\vec{W})_y + (\vec{N})_y = -\cos\alpha\, mg + N \Rightarrow N = mg\cos\alpha$$

$$\Rightarrow f = \mu\, mg\cos\alpha$$

$$\sum(\vec{F})_x = (\vec{f})_x + (\vec{W})_x = -\mu\, mg\cos\alpha + mg\sin\alpha = mg(\sin\alpha - \mu\cos\alpha) = \underline{8.353\ N}$$

c. What is the acceleration (magnitude and direction) of the box? (2 points)

Direction same as force, pos. x-direction.

$$a_x = \frac{1}{m}\sum(\vec{F})_x = g(\sin\alpha - \mu\cos\alpha) = \underline{3.213\ \tfrac{m}{s^2}}$$

d. If the incline is 5.5m long, how long will it take the box to slide down the incline all the way? (2 points)

$$x_f = \cancel{x_0}^{\,0} + \cancel{v_0 t_f}^{\,0} + \frac{1}{2} a_x t_f^2$$

$$\Rightarrow t_f = \sqrt{\frac{2x_f}{a_x}} = \underline{1.85\ s}$$

e. What is the work done by the friction as the box slides down the incline all the way? (2 points)

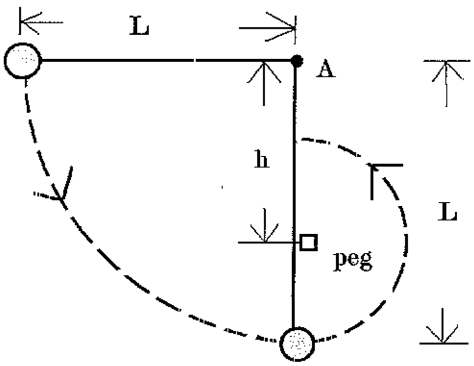
$\vec{f}$ = const. $\quad |\vec{f}| = \mu|\vec{N}| = \mu\, mg\cos\alpha =$

$\vec{d}$ = const. $\quad |\vec{d}| = 5.5\ m$

$\varphi_{\vec{f},\vec{d}} = 180°$

$$\Rightarrow W = \vec{F}\cdot\vec{d} = |\vec{f}||\vec{d}|\cos\varphi_{\vec{f}\vec{d}} = \underline{-34.44\ J}$$

4. **Energy conservation**



A ball is attached to a horizontal cord of length L whose other end is fixed at point A, see figure. Assume the force of gravity points vertically downward.

a) If the ball is released, what will be its speed at the lowest point of its path? Give your answer in terms of the cord length L. (2 points)

Let $U=0$ at the initial position.

$$K_1 + U_1 = K_2 + U_2$$

$$0 + 0 = \frac{1}{2} m v_2^2 - mgL \Rightarrow \underline{\underline{v_2 = \sqrt{2gL}}}$$

b) A peg is located a distance h directly below the point of attachment of the cord. If h=0.75 L, what will be the speed of the ball when it reaches the top of its circular path about the peg? Give your answer in terms of the cord length L. (2 points)

$$K_2 + U_2 = K_3 + U_3$$

diameter of smaller half-circle

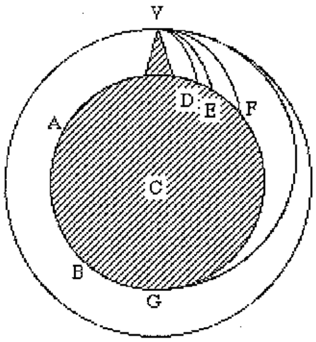
$$\frac{1}{2} m v_2^2 - mgL = \frac{1}{2} m v_3^2 - mg\left(L - \overbrace{2(L-h)}\right)$$

$$\Leftrightarrow mgL - mgL = 0 = \frac{1}{2} m v_3^2 - mgL\left(1 - 2\left(1 - \overbrace{\tfrac{3}{4}}^{1/2}\right)\right)$$

$$\Rightarrow \underline{\underline{v_3 = \sqrt{gL}}} < v_2 \quad ✓$$

5. **Newton's Cannon**

Suppose Earth had no atmosphere, and a ball were fired from the top of Mt Everest in a direction tangent to the ground. If the initial speed were high enough to cause the ball to travel in a circular trajectory around Earth, the ball's acceleration would be....



a) .... much less than g (because the ball doesn't fall to the ground)

b) be approximately g   Still close to the surface

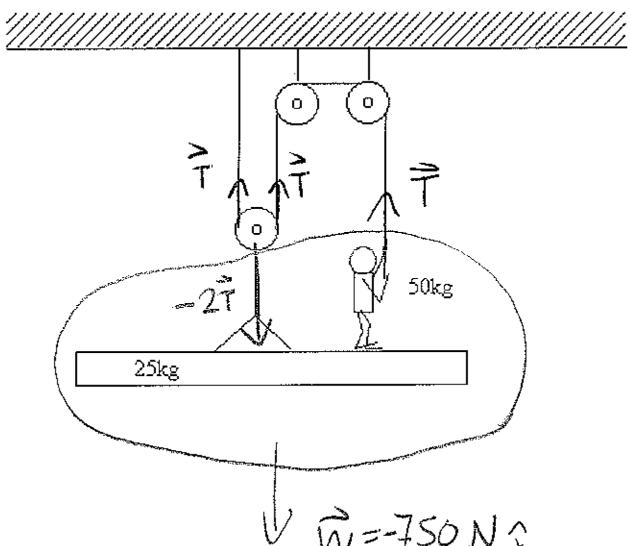
c) ....depend on the ball's speed

d) ... none of the above

(1 point)

6. **Person on Platform**

A 50kg person stands on a 25 kg platform. He pulls on the rope that is attached to the platform via the frictionless pulley system shown here. If he pulls the platform up at a steady rate, with how much force is he pulling on the rope? Ignore friction and assume $g=10m/s^2$. Circle the correct answer (1 point)



a) 750 N

b) 625 N

c) 500 N

d) 250 N

e) 75 N

f) 50 N

g) 25 N

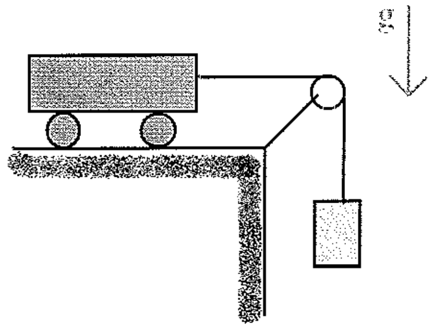
$\vec{W} = -750\,N\,\hat{y}$

$\sum(\vec{F})_y = 3T - 750N$

$\Rightarrow T = 250\,N$

7. Car lifting weight

A car is connected to a hanging weight by a string on a pulley (both massless). The car is initially moving to the left, lifting the mass before it stops and turns around. Circle the right answers. Ignore friction. (1 point each)



a. What happens to the kinetic energy of the car before it stops and turns around?

- Increases
- Decreases   Velocity gets smaller.
- Stays the same

b. What is the change in kinetic energy of the car before it stops and turns around?

- $\Delta K = 0$
- $\Delta K > 0$
- $\Delta K < 0$ $K_2 - K_1 < 0$ since $v_2 < v_1$
- Impossible to tell

c. What work does tension do on the car before it stops and turns around?

- None
- Positive work
- Negative work $\vec{V}$ $\vec{T}$ 180°
- Impossible to tell

d. What is the net work done on the car before it stops and turns around?

- Wnet = 0
- Wnet > 0 $\vec{N}$, $\vec{W}$ are perp. to displacement, do no work.
- Wnet < 0   Only tension does work.
- Impossible to tell