Physics 171 **PRINCIPLES OF PHYSICS I** **AQ 2005**

Final Exam

November 23, 2005

Name Key

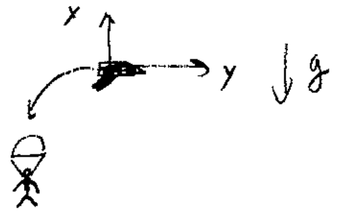
INSTRUCTIONS: Complete all problems. Show all work in the space provided. No books or notes allowed. Partial credit will be given for problems correctly set up even if incorrectly answered. No credit will be given for a numerical answer without supporting work. One point will be deducted from each answer without correct units. The time limit for this examination is 2 hours.

Good luck!

1. **Projectile Motion**

A plane with parachutists is flying horizontally at an altitude of 2500 m with a speed of 120 m/s. A parachutist jumps off the back of the plane with a horizontal velocity of 5 m/s (relative to the plane) against the direction of the plane.

a. Draw a diagram of the situation and choose a coordinate system. (1 point)



b. How long does it take the parachutist to reach an altitude of 700 m, where he opens his parachute? (2 points)

$a_y = -g \Rightarrow y = y_0 + v_{oy} t + \frac{1}{2}(g)t^2 = 0 + 0 - \frac{1}{2} g t^2 \stackrel{!}{=} -1800\,m$

$V_{oy} = 0$

$\Rightarrow t = \sqrt{\frac{2y_f}{g}} = \underline{\underline{19.17\,s}}$

c. How far did he move horizontally relative to the ground up to this point?(2 points)

$a_x = 0, \quad V_{ox} = 120\,\frac{m}{s} - 5\,\frac{m}{s} = 115\,\frac{m}{s}, \quad x_0 = 0$

$x(t) = x_0 + V_{ox} t = 0 + (115\,\frac{m}{s})(19.17\,s) = \underline{\underline{2200\,m}}$

d. What is his total velocity at this point? (Magnitude and direction) (2 points)

$V_x = \text{const.} = 115\,\frac{m}{s}$

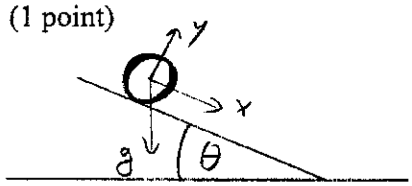
$V_y = V_0 + a_y t = (-9.8\,\frac{m}{s^2})(19.17\,s) = -187.9\,\frac{m}{s}$

$\Rightarrow |\vec{V}| = \sqrt{V_x^2 + V_y^2} = \underline{\underline{220\,\frac{m}{s}}}, \quad \tan\varphi = \frac{V_y}{V_x} \Rightarrow \underline{\underline{\varphi = -58.5^\circ}}$

2. **Pipe down incline**

A hollow pipe of mass 1.2 kg and radius 5 cm rolls down a~~n~~ frictionless incline of $\theta = 20°$. The pipe starts its motion at a distance $d = 2.8$ m from the base of the incline.

a. Draw the situation and free body diagram and indicate your choice of a coordinate system (1 point)



b. Calculate the potential energy at the starting point. (2 point)

$$U = mgh = mg\,d\sin\theta = (1.2\,kg)(9.8\,\tfrac{m}{s^2})(2.8\,m)\sin 20^\circ$$
$$= \underline{\underline{11.26\,J}}$$

c. What is the normal force on the pipe? (2 points)

$$\sum F_y = 0 \Rightarrow F_{g,y} + F_N = 0 \Rightarrow F_N = -F_{g,y} = mg\cos\theta = \underline{\underline{11.05\,N}}$$

d. Evaluate the moment of inertia ($I = M R^2$) of the pipe. (1 point)

$$I = (1.2\,kg)(0.05\,m)^2 = \underline{\underline{0.003\,kg\,m^2}}$$

e. What will the speed of the pipe be at the base of the incline? (*Hint: Write down the total energy of the system as a function of the linear velocity. Remember the relation between linear and angular velocity.*) (3 points)

$$E_i = E_f$$

$$\Leftrightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$$
$$= \frac{1}{2}\left(m + \frac{I}{R^2}\right)v^2$$

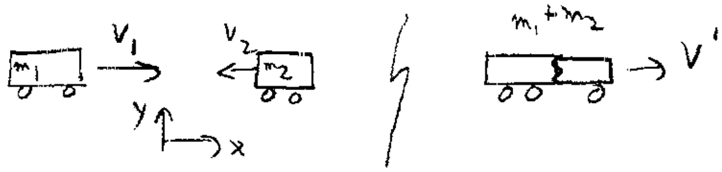
$$\Rightarrow v = \sqrt{2mgh \Big/ \left(m + \frac{I}{R^2}\right)}$$

$$= \sqrt{\frac{22.52\,J}{1.2\,kg + (0.003/0.05^2)\,kg}} = \underline{\underline{3.06\,\frac{m}{s}}}$$

3. **Collision**

Two cars crash head-on on an undivided highway. Obviously this is a totally inelastic collision. One car is traveling at 55 mph and has a mass of 900 kg, the other traveling at 40 mph has a mass of 1100 kg *(Hint: 1 mile=1609 m)*

a. Draw a schematic diagram illustrating the cars' velocities before and after the collision. (1 point)



b. Calculate the initial energy. (2 points)

$$E_i = K_1 + K_2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \underline{\underline{4.48\cdot 10^5\ J}}$$

$$v_1 = 55\ mph = 24.6\ \frac{m}{s}$$

$$v_2 = -40\ mph = -17.9\ \frac{m}{s}$$

c. What are the velocities of the cars after the collision? (2 points)

$v_1' = v_2' = v$, momentum conservation: $p_1 + p_2 = p'$

$$m_1v_1 + m_2v_2 = (m_1+m_2)v'$$

$$\Rightarrow v' = \frac{m_1v_1 + m_2v_2}{m_1+m_2} = \underline{\underline{1.225\ \frac{m}{s}}}$$

d. How much energy goes into deformation of the vehicles? *(Hint: calculate the kinetic energy after the collision)* (2 points)

$$K_f = \frac{1}{2}(m_1+m_2)(v')^2 = \underline{\underline{1500\ J}}$$

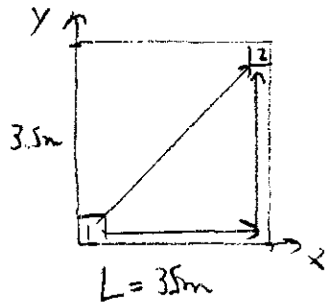
$$K_i = K_f + E_{deform} \Rightarrow E_{deform} = K_i - K_f = \underline{\underline{4.46\cdot 10^5\ J}}$$

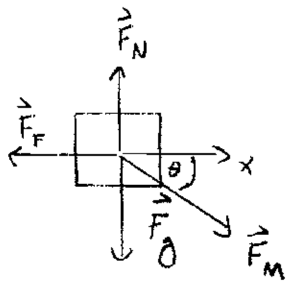
4. **Work**

of mass $m = 2.5$ kg

A man pushes a box from one corner of a square room (the sides have length 3.5m) to the distant corner at constant velocity. He is pushing the box at an angle of 45 degrees to the ground. The coefficient of kinetic friction is 0.2. with 120N

a. Draw a diagram of the physical situation with your choice of a coordinate system and also a free box diagram for the box. (2 points)





b. Calculate the normal force on the box. (2 points)

$$\sum F_y = 0 = F_{M,y} + F_N + F_g \Rightarrow F_N = -F_g - F_{M,y} = mg - F_{M,y}$$

$$= mg - |\vec{F}_M| \sin\theta = 109\text{ N}$$

c. Calculate the work done by friction on the box if it is pushed diagonally across the room. (2 points)

$$F_F = \mu_F F_N \;;\; W_F = \vec{F}\cdot\vec{d} = -F_F\sqrt{3.5^2 + 3.5^2}$$

$$= -108\text{ J}$$

d. Calculate the work done by the man on the box if it is pushed along the sides of the room to reach the distant corner. (2 points)

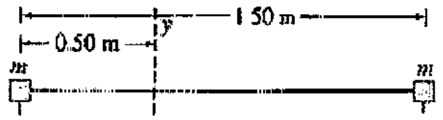
$$W = W_1 + W_2 = 2\,\vec{F}_M \cdot \vec{d} = 2$$

e. What is the total work done on the box in process d)? (1 point)

No net force $\Rightarrow W_{total} = 0$

5. **Moment of Inertia and Angular Momentum**

Consider the configuration of masses (m = 2.2 kg) shown below:



a. Calculate the moment of inertia of the system shown in the figure about the vertical axis (dashed line). (2 points)

$$I = \sum m_i r_i^2 = (2.2\,\mathrm{kg})\left((0.5\,\mathrm{m})^2 + (1\,\mathrm{m})^2\right) = \underline{2.75\,\mathrm{kg\,m^2}}$$

, with the right mass coming out of the plane of the paper initially

b. Determine the angular momentum of the system around this axis if it is spinning at 25 rpm (magnitude and direction). (2 points)

$\vec{L} = I\vec{\omega}$; $\vec{L}$ points in direction of $\vec{\omega}$, i.e. negative y-axis

$$\omega = 25\,\mathrm{rpm} \cdot 2\pi = \frac{50\pi}{60}\frac{1}{s} = 2.62\,\mathrm{Hz}$$

$$L = \underline{7.205\,\mathrm{kg}\,\frac{m^2}{s}}$$

c. Evaluate the kinetic energy of the system. (2 points)

$$K_{rot} = \frac{1}{2} I \omega^2 = \frac{1}{2}(2.75\,\mathrm{kg\,m^2})\left(2.62\,\frac{1}{s}\right)^2$$

$$= \underline{9.44\,\mathrm{J}}$$