Physics 171 PRINCIPLES OF PHYSICS I AQ 2005

Second Midterm Exam

November 4, 2005

Name Key

INSTRUCTIONS: Complete all problems. Show all work in the space provided. No books or notes allowed. Partial credit will be given for problems correctly set up even if incorrectly answered. No credit will be given for a numerical answer without supporting work. One point will be deducted from each answer without correct units. The time limit for this examination is 1 hour and 20 minutes.

Good luck!

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1. **Vector Multiplication**

Consider two vectors, **A**= 4.7 **i** – 2.1 **j** , and **B** = 4.2 **i** – 5.4 **j** + 1.3 **k**.

a. Calculate their scalar product. (2 points)

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z = (4.7)(4.2) + (-2.1)(-5.4) + (0)(1.3)$$

$$= \underline{\underline{31.08}}$$

b. What is the angle between the two vectors? (2 points)

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta \Rightarrow \cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = 0.867$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} = 5.148$$

$$|\vec{B}| = 6.963$$

$$\Rightarrow \theta = \underline{\underline{29.89^\circ}}$$

2. **Gravitation**

Jupiter's moon Europa (mass 4.8×10^{22} kg) takes 3.551 days to go once around Jupiter while the latter circles the sun. In its orbit it sits at a distance of 671,034 km from Jupiter. The gravitational constant is $G=6.67\times10^{-11}\ Nm^2/kg^2$.

a. Calculate the centripetal force that Jupiter exerts on Europa to keep it in orbit. Assume a circular motion. (2 points)

$$F_c = m\frac{v^2}{r} = m\frac{\left(\frac{2\pi r}{T}\right)^2}{r} = m\frac{4\pi^2 r}{T^2} \quad ; \quad T = (3.551\ \cancel{d})\left(\frac{24\ \cancel{h}}{\cancel{d}}\right)\left(\frac{3600\ s}{\cancel{h}}\right)$$

$$= \underline{\underline{1.35\cdot10^{22}\ N}} \qquad\qquad = 306{,}806\ s$$

b. Equating this force with the force due to Newton's law of universal gravity, calculate the mass of Jupiter. (2 points)

$$F_c \overset{!}{=} F_g = G\frac{mM}{r^2} \Rightarrow M = \frac{r^2}{mG}F_c = \underline{\underline{1.899\cdot10^{27}\ kg}}$$

778.4 million km

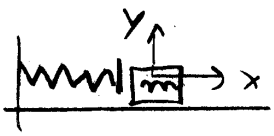
c. Calculate the potential energy of Jupiter in the gravitational field of the sun (mass 1.99×10^{30} kg) when the Jupiter is ~~1.49×10^{11}~~ m away from the center of the sun. Assume the potential energy to be zero at infinite distance from the sun. (2 points)

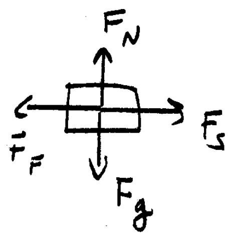
$$U = -G\frac{mM}{r} = \underline{\underline{-3.24\cdot10^{35}\ J}}$$

3. **Box and spring with friction**

A box of mass 1.7 kg, initially at rest, sits in front of a spring of stiffness k=95 N/m compressed by 45 cm. As the spring is released, it accelerates the box to the right on a plane with friction coefficient μ=0.1.

a. Draw a free body diagram indicating all the forces acting on the box and indicate your choice of a coordinate system. (1 point)





b. What is the normal force (magnitude and direction) acting on the box? (1 point~~s~~)

$F_N = mg = 16.66\ N$; perpendicular to plane, +y direction

c. What is the work done by friction on the box? (2 points)

$W = \vec{F}\cdot\vec{d}$, since $\vec{F}$, $\vec{d}$ are const.

$= -\mu_k m g\ (1.65m) = -2.75\ J$

$1.2m + 0.45m$

d. What is the velocity of the box after it traveled 1.2 m away from the spring's equilibrium point? (3 points)

$E_B = E_A + W = \frac{1}{2}kx^2 + W_F$, $x = 5cm = 0.05m$

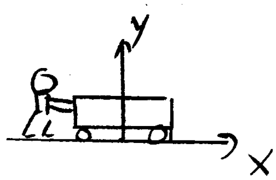
$\stackrel{!}{=} \frac{1}{2} m v_B^2$ $\Rightarrow$ $v_B = \sqrt{\frac{k}{m}x^2 + \frac{2}{m}W_F}$

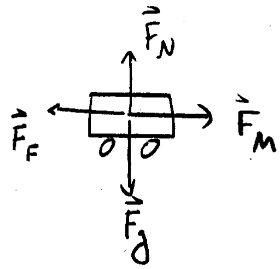
$= 2.84\ \frac{m}{s}$

4. **Work and Energy**

A person is trying to push-start a car of mass 950 kg. Pushing it, the force that he exerts on the car is parallel to the ground. He is able to move the car over a distance of 7.4 m at constant velocity, before its engine finally starts. The effective coefficient of kinetic friction is μ=0.24.

a. Draw a free body diagram indicating all forces acting on the car and indicate your choice of a coordinate system. (1 point)





b. Calculate the force exerted by the man on the car. (3 points)

$$\Sigma F = 0 \;, \quad \text{since} \quad \Sigma F_y = F_N + F_{g,y} = 0 \;\Rightarrow\; F_N = -F_{g,y} = mg$$

$$\text{also} \quad \Sigma F_x = F_M + F_F = 0 \;; \quad |\vec{F}_F| = |\vec{F}_N| \mu_k :$$

$$F_M = -F_F = -(-\mu_k mg) = (0.24)(950\,kg)(9.8\,\tfrac{m}{s^2}) = \underline{\underline{2234\,N}}$$

neg. x direction

c. Determine the work done by ~~the man~~ friction on the car until the motor starts. (2 points)

$$W_f = \vec{F}_{\cancel{M}} \cdot \vec{d} = (-2234\,N)(7.4\,m) = \underline{\underline{-16{,}530\,J}}$$

$$F_M = -F_F$$

d. What is the total work done on the car? (1 point)

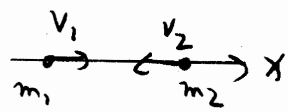
$$F_{net} = \Sigma F = 0 \;\Rightarrow\; W_{net} = \int F_{net} = 0$$

5. **Collision**

A ball of mass ~~0.260 kg~~ ~~0.15~~ 150 g and speed 2.30 m/s collides head-on with a ~~0.410 kg~~ 320 g ball.

a. Assume a totally inelastic collision and the second ball moving in opposite direction as the first ball with a speed of 2.90 m/s. Determine the velocities of the balls after the collision. (2 points)

Inelastic collision: $v_1' = v_2' = v'$



$$\sum p_i = \sum p_f$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v' \Rightarrow v' = \frac{1}{m_1 + m_2}(m_1 v_1 + m_2 v_2)$$

$$= \frac{1}{0.470\,kg}\left(0.15\,kg\ 2.3\,\tfrac{m}{s} - 0.32\,kg\ 2.9\,\tfrac{m}{s}\right)$$

b. In which direction are the balls moving after the collision? (1 point)

$$= -1.24\,\frac{m}{s}$$

In the neg. x direction (i.e. in the direction of the second ball)

c. Now assume instead an elastic collision ~~and~~, the second ball initially at rest, and $m_1 = m_2 = 150\,g$. Calculate the velocities of the balls after the collision. (2 points)

$v_2 = 0$, $v_1' = ?$, $v_2' = ?$

Use $\sum p_i = \sum p_f$ and $\sum K_i = \sum K_f$

$$m_1 v_1 = m_1 v_1' + m_2 v_2'$$

$$\tfrac{1}{2} m_1 v_1^2 = \tfrac{1}{2} m_1 v_1'^2 + \tfrac{1}{2} m_2 v_2'^2$$

$$\Rightarrow v_2' = \frac{m_1}{m_2}(v_1 - v_1') = v_1 - v_1'$$

$$\Rightarrow v_1^2 = v_1'^2 + v_2'^2 = v_1'^2 + (v_1 - v_1')^2$$

$$\cancel{v_1^2} = v_1'^2 + \cancel{v_1^2} - 2 v_1 v_1' + v_1'^2$$

$$\Rightarrow v_1' = v_1 \text{ or } v_1' = 0$$

$$\Rightarrow \underline{\underline{v_1' = 0 \Rightarrow v_2' = v_1}}$$