Physics 171 PRINCIPLES OF PHYSICS I AQ 2005

First Midterm Exam

October 14, 2005

Name Key

INSTRUCTIONS: Complete all problems. Show all work in the space provided. No books or notes allowed. Partial credit will be given for problems correctly set up even if incorrectly answered. No credit will be given for a numerical answer without supporting work. One point will be deducted from each answer without correct units. The time limit for this examination is 1 hour and 20 minutes. **Good luck!**

---

1. **Vector Addition**

Given are the vectors **A** and **B** with the length A=23.0 units, B=58.6 units and the angles φ_A= 46.0° and φ_B= 140°.

a. Calculate the Cartesian components of the two vectors. (2 points)

$A_x = |\vec{A}| \cos\varphi_A = 15.977$; $B_x = -44.890$

$A_y = |\vec{A}| \sin\varphi_A = 16.545$; $B_y = 37.667$

b. Calculate the length & direction of **A+B** and **A-B** ~~and express the result in polar coordinates~~. (4 points)

$\vec{A}+\vec{B} = \vec{C} = \begin{pmatrix} -28.913 \\ 54.212 \end{pmatrix}$; $\vec{A}-\vec{B} = \vec{D} = \begin{pmatrix} 60.867 \\ -21.122 \end{pmatrix}$

$\Rightarrow |\vec{C}| = \sqrt{C_x^2 + C_y^2} = 61.440$; $|\vec{D}| = 64.428$

$\tan\varphi_C = \frac{C_y}{C_x}$ $\qquad \varphi_D = -19°$

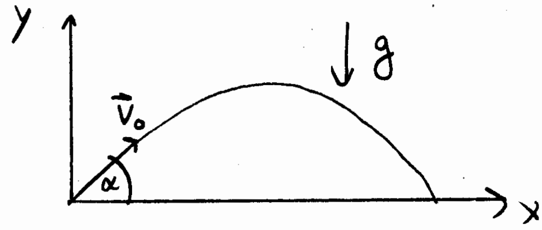
$\Rightarrow \varphi_C = 118°$ (see part c)

c. Draw a diagram of the vectors **A, B, A+B** and **A-B** on the sheet provided. (2 points)

2. **Projectile Motion**

A golfer tries to put a golf ball into a hole with a direct shot. He gives the ball an initial velocity of v_0=22 m/s with an angle 25° from the ground.

a. Draw a diagram of the situation and indicate your choice of a coordinate system. (1 point)



b. Write down the relevant equations describing the motion of the ball in horizontal and vertical direction and indicate the known and unknown quantities (3 points)

$x(t) = x_{0x} + v_{0x} t + \frac{1}{2} a_x t^2$

$x_0 = 0$; $v_{0,x} = |\vec{v}_0| \cos \varphi_{v_0}$

$= 19.94 \frac{m}{s}$

$a_x = 0$

Unknown: x, t

$y(t) = y_0 + v_{0,y} t + \frac{1}{2} a_y t^2$

$y_0 = 0$; $v_{0,y} = |\vec{v}_0| \sin \varphi_{v_0} = 22 \frac{m}{s} \sin 25°$

$= 9.30 \frac{m}{s}$

$a_y = -g$

Unknown: y, t

c. What maximal altitude will the ball reach? (3 points)

At maximal altitude, we have $v_y = 0$ ⇒ Use $v_y^2(t) = v_{0,y}^2 + 2a_y(y - y_0)$

$\Rightarrow 0 = v_{0y}^2 - 2g\, y_{max} \Rightarrow y_{max} = \frac{v_{0,y}^2}{2g} = \underline{\underline{4.4\ m}}$

d. How long will it take the ball to hit the ground again? (3 points)

At that time: $y(t_{final}) = 0$

$\Rightarrow 0 = y_0 + v_{0,y} t + \frac{1}{2} a_y t^2 \Rightarrow 0 = v_{0,y} t - \frac{1}{2} g t^2$

$\Rightarrow t = 0$ or $t = \frac{2v_{0,y}}{g} = \underline{\underline{1.9\ s}}$

e. What is the velocity (both components!) at the highest point? (2 points)

$v_x = \text{const.} = \underline{\underline{v_0}}$

$v_y = v_{0,y} + at = v_{0,y} - g\left(\frac{1.9s}{2}\right) = \underline{\underline{0}}$

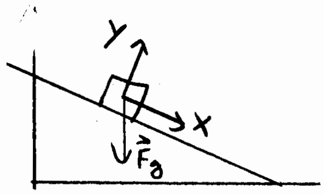
f. As a hoax, the golfer also hits a baseball with a mass 7 times larger than the golf ball with the same kind of swing. Which parts of the calculation will change? (1 point)

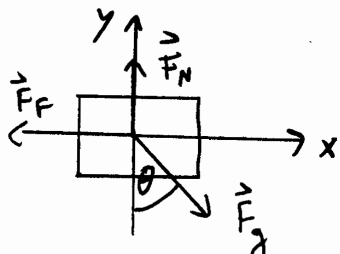
Nothing.

3. **Box on incline**

A box of mass 2.3 kg, initially at rest, slides down an incline with an angle of 32° with a coefficient of kinetic friction of μ_k=0.15.

a. Draw a free body diagram indicating all the forces acting on the box and indicate your choice of a coordinate system. (1 point)





b. What is the acceleration (magnitude and direction) of the box? (2 points)

$$a_x = \frac{1}{m}\sum F_x = F_{g,x} + F_{F,x} = \frac{1}{m}\left(mg\cos(270°+\theta) - \mu_k F_N\right)$$

$$= g\cos(270°+\theta) + \mu_k g\sin(270°+\theta)$$

$$= \underline{\underline{3.95\ \frac{m}{s^2}}}$$

c. What is the normal force acting on the box? (2 points)

$\sum F_y = 0$, since motion is along the plane (or x-direction)

$$\Rightarrow \quad = F_{g,y} + F_{N,y} \Rightarrow F_{N,y} = -F_g = -mg\sin(270°+\theta) = +\underline{\underline{19.1\ N}}$$

d. If the incline is 6.8m long, how long will it take the box to slide down the incline all the way? (2 points)

$$x(t) = x_0 + v_{0,x}t + \frac{1}{2}a_x t^2$$

$$\Rightarrow x(t_{final}) = 6.8m = 0 + 0t + \frac{1}{2}\left(3.95\ \frac{m}{s^2}\right)t^2$$

$$\Rightarrow t^2 = \frac{2x_{final}}{a_x} \Rightarrow t = \underline{\underline{1.86s}}$$

e. What is the net force (magnitude and direction) that causes the box to slide down the incline? (2 points)

$$\vec{F}_{Net} = \sum_{\text{all forces}} \vec{F} = m\vec{a} = m\begin{pmatrix} a_x \\ 0 \end{pmatrix} = \begin{pmatrix} 9.08N \\ 0 \end{pmatrix}$$

$$\Rightarrow |\vec{F}_{net}| = 9.08N, \quad \varphi_{F_{net}} = 0° \quad \text{(in x direction)}$$

4. **Circular Motion**

Saturn's brightest moon Titan was recently explored by the space probe "Huygens". It travels around the giant planet in a (nearly) circular orbit, and is 1,221,830 km away from Saturn's center.

a. Calculate the acceleration that Titan is experiencing if its period around Saturn is T=15.95 days. *(Hint: Remember how velocity is defined in terms of the elapsed time)*

(2 points)

$$a_R = \frac{v^2}{r} = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r}{T^2} \quad ; \quad 15.95\,d = 1{,}378{,}080\,s$$

$$= \underline{\underline{0.025\,\frac{m}{s^2}}}$$

b. Titan has a mass of 1.35×10^{23} kg. What force does Saturn have to exert on it to keep it in orbit? What is the direction of this force? (2 points)

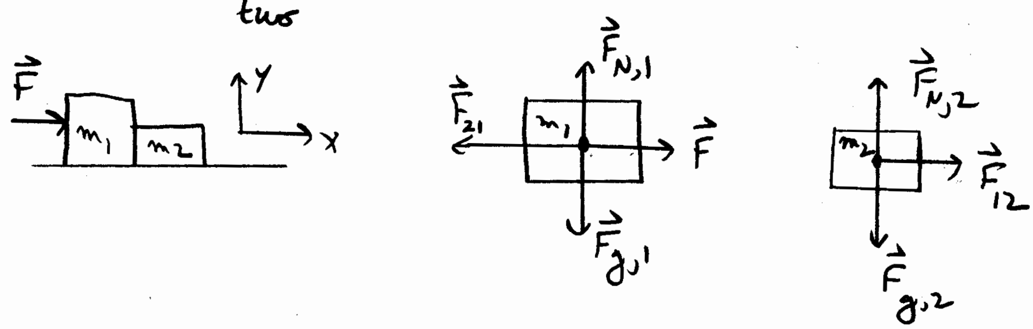
$$F_c = m a_R = \underline{\underline{3.429 \cdot 10^{21}\,N}}$$

(directed towards Saturn's center)

5. **Two boxes in contact**

~~Three~~ Two boxes in contact with each other are placed on a table. A force **F** is exerted on the leftmost box to the right (positive x direction). Assume no friction is present.

a. Draw a diagram of the situation and ~~3~~ 2 individual free body diagrams indicating the forces acting on each of the ~~three~~ two boxes. (3 points)



b. Calculate the acceleration of each ~~of the~~ two boxes algebraically. (Hint: the boxes will move simultaneously) (2 points)

$$\Sigma F = ma \Rightarrow \underset{\text{forces on } m_1}{\Sigma F} = m_1 a_1 \,. \quad \text{Boxes are on table} \Rightarrow \Sigma F_{1,y} = \Sigma F_{2,y} = 0$$

$$\text{Boxes will move together} \Rightarrow a_{1x} = a_{2x} = \frac{1}{m_1+m_2} \Sigma F = \frac{F}{m_1+m_2}$$

c. Calculate the net force on the first box algebraically. (2 points)

$$\Sigma F_{on\,1} = m_1 a_1 = \frac{m_1 F}{m_1+m_2}$$

d. Evaluate the expressions from ~~a~~ b and ~~b~~ c numerically if the masses of the two boxes are 1.2 kg and 4.4 kg, respectively, and the force on the leftmost box is 98 N to the right (+x direction). (1 point)

$$a_{1x} = a_{2x} = \frac{98\,N}{m_1+m_2} = \frac{98\,N}{5.6\,kg} = \underline{17.5\,\frac{m}{s^2}}$$

$$\Sigma F_{a1} = F_{net,1} = \underline{\underline{21\,N}}$$