

First Midterm Exam

October 9, 2009

Name Key

INSTRUCTIONS: Complete all problems. Show all work in the space provided. No books or notes allowed. Partial credit will be given for problems correctly set up even if incorrectly answered. No credit will be given for a numerical answer without supporting work. One point will be deducted from each answer without correct units. The time limit for this examination is 1 hour and 20 minutes. **Good luck!**

Constants: $g = 9.8 \text{ m/s}^2$ $1 \text{ mi} = 1602 \text{ km}$ $1 \text{ m} = 3.28 \text{ ft}$ $1 \text{ yard} = 3 \text{ ft}$ $1 \text{ in} = 2.54 \text{ cm}$

1 Vector Addition

Consider a vector \mathbf{A} with length $A=7.5$ units at an angle of 225 degrees with respect to the positive x axis, and a vector $\mathbf{B} = -6.7 \mathbf{i} - 4.8 \mathbf{j}$.

- a. Calculate the length and direction of the vector $\mathbf{C} = 5\mathbf{A} - 2\mathbf{B}$. (3 points)

$$A_x = |\vec{A}| \cos \varphi_A = -5.303 \quad ; \quad B_x = -6.7$$

$$A_y = |\vec{A}| \sin \varphi_A = -5.303 \quad ; \quad B_y = -4.8$$

$$\left. \begin{aligned} C_x &= 5A_x - 2B_x = -13.115 \\ C_y &= 5A_y - 2B_y = -16.915 \end{aligned} \right\} \text{III. Quadrant}$$

$$|\vec{C}| = \sqrt{C_x^2 + C_y^2} = 21.404$$

$$\tan \varphi_c = \frac{C_y}{C_x} \Rightarrow \varphi_c = 52.2^\circ \quad (232.2^\circ)$$

- b. Find the vector \mathbf{D} that you would have to add to $3\mathbf{B}$ to obtain a 5.2 unit-long vector in negative x-direction? (3 points)

$$\text{Need } 3\vec{B} + \vec{D} = -5.2 \hat{i} \quad \approx \quad \begin{aligned} 3B_x + D_x &= -5.2 \\ 3B_y + D_y &= 0 \end{aligned}$$

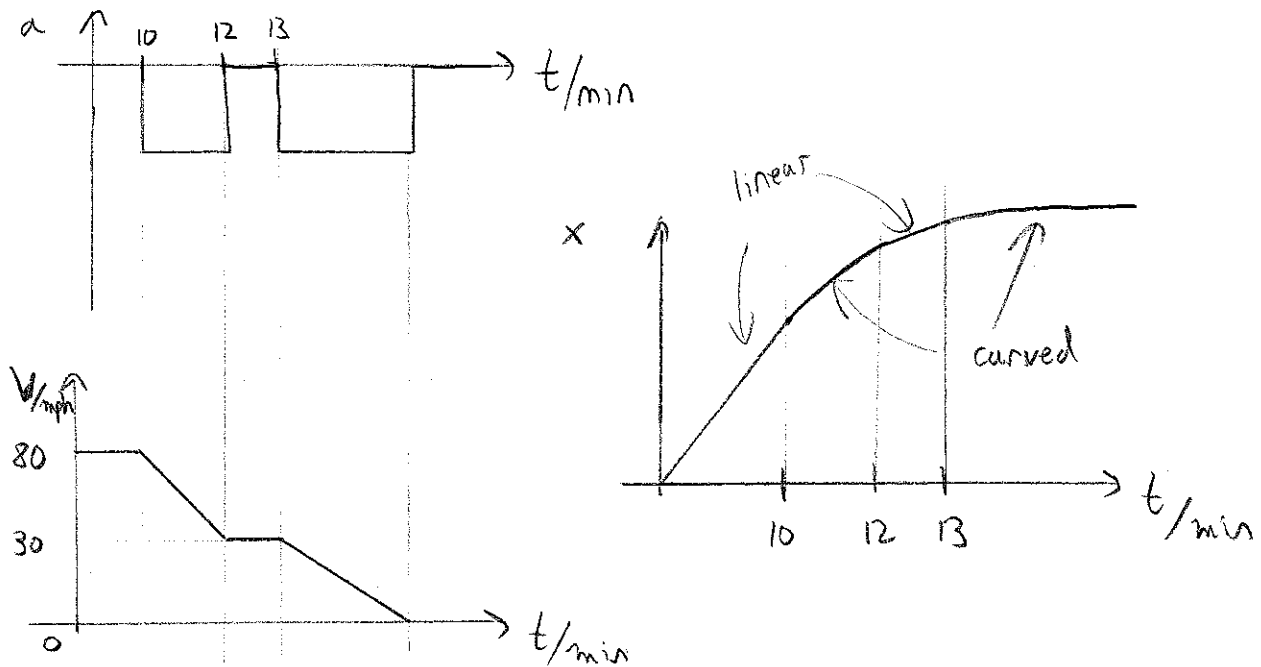
$$\Rightarrow D_y = -3B_y = +14.4$$

$$D_x = -5.2 - 3B_x = 14.9$$

2. Motion Diagrams

A train rolls down the tracks at 85 mph between two cities 320 miles apart. Ten minutes into the trip the conductor is informed that the track is damaged and that he has to stop the train as quickly as possible. He is able to slow down to 30 mph in 2 minutes with constant acceleration. He then travels at constant speed ^{x for a minute} to let the brakes cool off before braking with the same constant acceleration as before and comes to a complete stop.

a. Draw all three motion diagrams showing the kinematic variables of the ^{train} car. The diagrams can be qualitative (3 points)



b. Calculate the acceleration of the train while it's braking. Your result should be in SI units. (2 points)

$$a = \frac{\Delta V}{\Delta t} = \frac{30 \text{ mph} - 85 \text{ mph}}{2 \text{ min}} = \frac{-55 \text{ mph}}{120 \text{ s}} = -\frac{55}{120} \frac{\text{mph}}{\text{s}} \left(1602 \frac{\text{m}}{\text{mi}} \right) \left(\frac{1}{3600} \frac{\text{h}}{\text{s}} \right) = -0.204 \frac{\text{m}}{\text{s}^2}$$

c. When does the train come to a complete stop? (2 points)

Need to stop from 30 mph with acceleration calculated above.

$$v = v_0 + at \Rightarrow t = \frac{v - v_0}{a} = \frac{-30 \text{ mph}}{-55 \text{ mph}} \cdot 120 \text{ s} = 65.45 \text{ sec.}$$

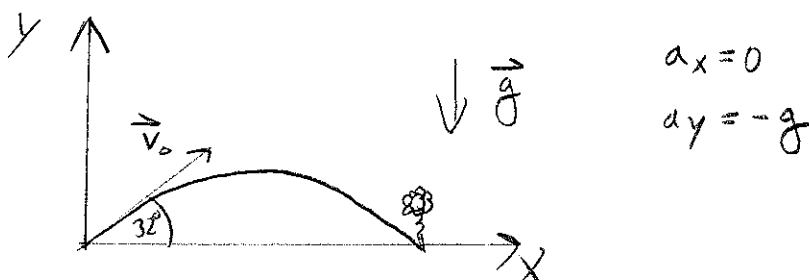
(total stopping time 2 min + 1 min + 65s)
 $\approx 4 \text{ min.}$

3. Projectile Motion

A gardener is trying to water a rose in a flower bed. The water leaves the garden hose with a velocity of 20 m/s, and he aims at an angle 32° with respect to the ground. Assume hose end and rose are at the same height.

- a. Draw a diagram of the situation and indicate your choice of a coordinate system.

(1 point)



- b. What maximal altitude will the water reach?

(2 points)

$$v_{x0} = 16.96 \text{ m/s}$$

$$v_{y0} = 10.60 \text{ m/s}$$

$$v_{y \text{ highest}}^2 = v_{y0}^2 + 2a_y(y_{\text{highest}} - y_0) \Rightarrow y_{\text{highest}} = -\frac{v_{y0}^2}{2a_y} = +\frac{v_{y0}^2}{2g} = \underline{\underline{5.73 \text{ m}}}$$

- c. How far away from the end of the hose is the rose located?

(3 points)

Need to know time allowed:

$$y_f = y_0 + v_{y0} t_f + \frac{1}{2} a_y t_f^2 \quad ; \quad y_0 = y_f = 0 \Rightarrow t_f = 0 \text{ or}$$

$$\Rightarrow x_f = v_{x0} t_f = \underline{\underline{36.63 \text{ m}}} \quad t_f = \frac{2v_{y0}}{-a_y} = \frac{2v_{y0}}{g} = 2.16 \text{ s}$$

- e. What are the velocity and the acceleration (both components of both!) of the water at the highest point on its trajectory?

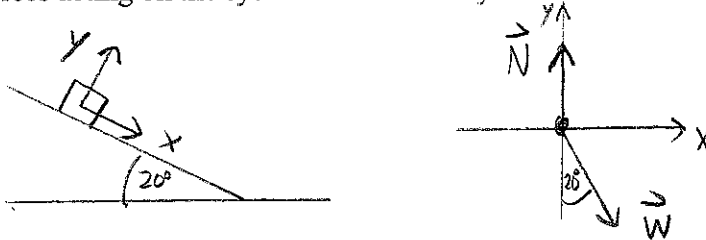
(2 points)

$$\vec{v}_{\text{highest}} = \begin{pmatrix} v_{x0} \\ 0 \end{pmatrix} \quad ; \quad \vec{a}_{\text{highest}} = \vec{a} = \begin{pmatrix} 0 \\ -g \end{pmatrix}$$

4. **Box on Incline**

A box of mass 3 kg slides down a fixed inclined plane of angle 20° without friction. It starts at rest 7 m upwards of the base of the plane.

- a. Draw a pictorial representation of the situation and a free body diagram indicating all the forces acting on the system and indicate your choice of a coordinate system. (2 points)



- b. What is the normal force acting on the box? (2 points)

$$\sum_{i=1}^2 (\vec{F}_i)_y = m a_y = 0 = (\vec{N})_y + (\vec{W})_y = |\vec{N}| - mg \cos 20^\circ$$

$$\Rightarrow |\vec{N}| = mg \cos 20^\circ = \underline{\underline{27.6 \text{ N}}}$$

- c. What is the acceleration (magnitude and direction or Cartesian coordinates) of the box? (2 points)

$$\sum_{i=1}^2 (\vec{F}_i)_x = m a_x = 0 + (\vec{W})_x = mg \sin \theta$$

$$= 10.1 \text{ N} \Rightarrow a = \frac{F}{m} = \underline{\underline{3.35 \frac{\text{m}}{\text{s}^2}}}$$

Direction: down the incline, i.e. pos. x.

- d. Calculate the speed of the box down the plane after 1.5 seconds. (2 points)

$$v_x = v_{x0} + a_x t$$

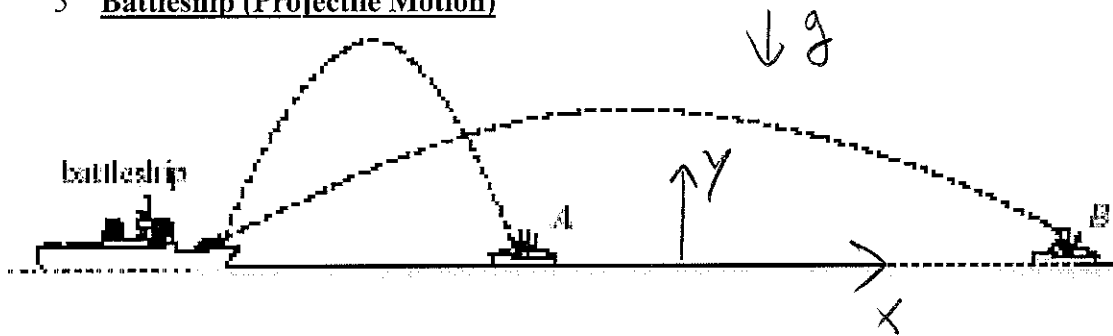
$$\Rightarrow v_x(t=1.5 \text{ s}) = 0 + (3.35 \frac{\text{m}}{\text{s}^2})(1.5 \text{ s}) = \underline{\underline{5.03 \frac{\text{m}}{\text{s}}}}$$

- e. How long does it take the box to reach the base of the incline? (2 points)

$$x_f = x_0 + v_{x0} t_f + \frac{1}{2} a_x t_f^2$$

$$\Rightarrow t_f = \sqrt{\frac{2x_f}{a_x}} = \sqrt{\frac{2 \times 7 \text{ m}}{3.35 \frac{\text{m}}{\text{s}^2}}} = \underline{\underline{2.04 \text{ s}}}$$

5 Battleship (Projectile Motion)



A battleship fires two shells towards enemy ships A & B.

A. Which one gets hit first? Circle the right answer. (1 point)

- a. A
- b. B *Takes longer time to reach higher altitude as required for shell hitting A.*
- c. Both get hit at the same time.
- d. Not enough information

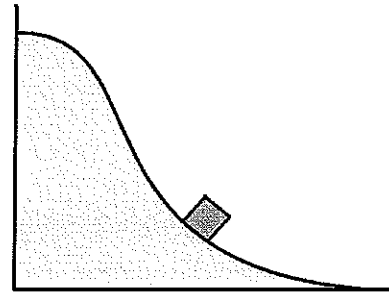
B. Which statement is true about the velocities of the shells **as they are about to** hit A and B? Circle the right answer. (1 point)

- a. Vertical component of the shell hitting A is bigger than of that hitting B.
- b. Horizontal component of shell hitting A is bigger than of that hitting B.
- c. The horizontal components of the shells are equal just before they hit the ships.
- d. The vertical components of the shells are equal just before they hit the ships
- e. None of the above. *Steeper descent of shell A, so final velocity in y-direction more negative, i.e. smaller than of shell B.*

6. Car on roller coaster

A car on a roller coaster rolls down the track shown on the right. As the cart rolls beyond the point shown, what happens to its speed and acceleration in the direction of motion?

Circle the correct answer. (1 point)



- a. Both decrease
- b. The speed decreases, but the acceleration increases.
- c. Both remain constant.
- d. The speed increases but the acceleration decreases
- e. Both increase
- f. None of the above

• Slope smaller \rightarrow less acceleration
• Acceleration positive \rightarrow velocity increases

7. Boxes on a table

Three boxes A, B, C of mutually different masses are sitting in contact with one another on a frictionless table. There is a force on the left box (A) pushing it to the right. Which is a correct statement about the contact forces that each box exerts on its neighbor?

(1 point)

- a. The forces between all the boxes are of equal strength and direction.
- b. The forces between all the boxes are of equal strength.
- c. The force of box A on box B is the same as the force of box B on box C.
- d. The force of box A on box B is the same as the force of box C on box A.

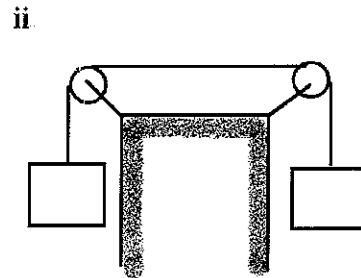
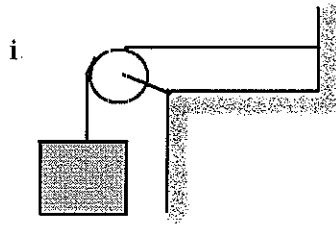
e. None of the above.

$$F_{AB} = F \frac{m_B + m_C}{m_A + m_B + m_C} \neq F_{BC} = F \frac{m_C}{m_A + m_B + m_C}$$

8. Tension

Compare the tension of the rope in the following situations (see diagram). Circle the right answer. (1 point)

- i. A mass of 5kg hangs vertically from a rope that is redirected by a massless pulley and then attached to a wall.
- ii. Instead of being attached to the wall, the rope is attached to another mass of 5kg via a second massless pulley.



- a. The tension in situation i. is bigger than in situation ii.
- b. The tension in situation i. is smaller than in situation ii.
- c. The tension is the same in both situations.

Wall or other weight have to hold up 5kg mass.