

Second Midterm Exam

February 25, 2009

Name Key

INSTRUCTIONS: Complete all problems. Show all work in the space provided. No books or notes allowed. Partial credit will be given for problems correctly set up even if incorrectly answered. No credit will be given for a numerical answer without supporting work. One point will be deducted from each answer without correct units. The time limit for this examination is 1 hour and 20 minutes. **Good luck!**

Constants

$$\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

$$m_p = 1.673 \times 10^{-27} \text{ kg}$$

$$m_e = 9.110 \times 10^{-31} \text{ kg}$$

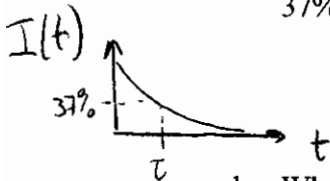
$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

1. RC circuit

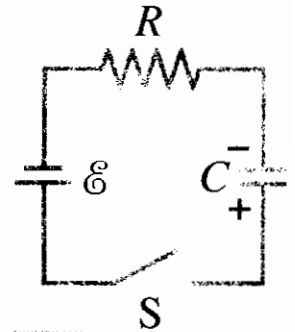
Consider the RC circuit on the right with $R = 1 \text{ k}\Omega$, $C = 22 \text{ pF}$, and emf 12 V .

- a. How long after the switch has been closed will the current be at 37% of its maximal value? (1 point)



$$\tau = RC = (10^3 \Omega)(22 \cdot 10^{-12} \text{ F}) = 22 \cdot 10^{-9} \text{ s}$$

$$= \underline{\underline{22 \text{ ns}}}$$



- b. What is the voltage across the resistor at this point in time? (2 points)

$$V_c(t=\tau) = V_0(1 - e^{-\tau/\tau}) = V_0(1 - \frac{1}{e}) ; V_0 = \frac{Q_{\text{max}}}{C} = 12 \text{ V}$$

$$= 12 \text{ V} (1 - \frac{1}{e}) = \underline{\underline{7.59 \text{ V}}}$$

$$\mathcal{E} - V_R - V_C = 0 \Rightarrow V_R = \mathcal{E} - V_C = \underline{\underline{4.41 \text{ V}}}$$

- c. What is the maximal charge on the capacitor? (2 points)

$$Q_{\text{max}} = CV = (22 \text{ pF})(12 \text{ V}) = \underline{\underline{2.64 \cdot 10^{-10} \text{ C}}}$$

2. Particle in magnetic field

Two perpendicular wires are separated by 4 mm. Each wire has a current of 15 A in the directions indicated. A proton p midway between the wires is moving with a speed of 4×10^6 m/s parallel to the top wire from left to right.

_____ $I_1 = 15\text{A} \rightarrow (+x \text{ direction})$

$p \rightarrow v = 4 \times 10^6$ m/s (+x direction, separation of wires in z direction)

$\otimes I_2 = 15\text{A}$ (I_2 flows into the page, +y direction)

- a) What is the magnetic field (magnitude and direction) due to the current in the top wire at the position of the proton? (2 points)

$$B = \frac{\mu_0}{2\pi} \frac{I}{r} = 2 \cdot 10^{-7} \frac{\text{Tm}}{\text{A}} \frac{15\text{A}}{4 \cdot 10^{-3}\text{m}} = 15 \cdot 10^{-4} \text{T}$$

Direction: into the page

- b) What is the magnetic field (magnitude and direction) due to the current in the bottom wire at the position of the proton? (2 points)

Same strength as in a), direction to the right (+x)

- c) What is the **total** magnetic field (magnitude and direction) due to the currents in the wires at the position of the proton? (2 points)

$$|\vec{B}_{\text{total}}| = |\vec{B}_1 + \vec{B}_2| = \sqrt{2} |\vec{B}_1|, \text{ since } |\vec{B}_1| = |\vec{B}_2|$$

$$= 21.2 \times 10^{-4} \text{T}$$

Direction in xy plane, 45° from pos. x axis.

- d) What is the force (magnitude and direction) on the proton due to the magnetic field? (2 points)

$$\vec{F} = q \vec{v} \times \vec{B}_1; |\vec{F}| = e (4 \cdot 10^6 \frac{\text{m}}{\text{s}}) (15 \cdot 10^{-4} \text{T})$$

$$\text{since } \vec{B}_2 \parallel \vec{v} = 9.61 \times 10^{-16} \text{N}$$

Direction: up, +z direction

4. Torque on Hydrogen Atom

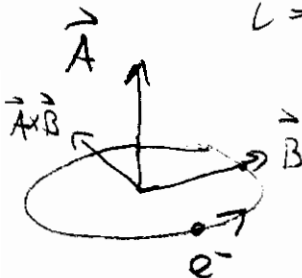
The hydrogen atom can be regarded as a tiny magnetic dipole, due to the current constituted by the electron whipping in a circle around a heavy nucleus.

a. Calculate the magnetic dipole moment of the hydrogen atom assuming, as in the Bohr model, that it is in its ground state of radius 0.0529 nm. (Hint: Current is the charge passing a point per unit time. Obtain the velocity of the electron by using $ma = mv^2/r$ and the electrostatic force between electron and proton: $F = qQ/4\pi\epsilon_0 r^2$.) (2 points)

$$\begin{aligned} \mu &= NIA \quad ; \quad A = \pi r^2 \quad , \quad N = 1 \\ I &= \frac{e}{T} \quad ; \quad T = \frac{2\pi r}{v} \quad ; \quad \frac{e^2}{4\pi\epsilon_0 r^2} = \frac{mev^2}{r} \\ &= \frac{ev}{2\pi r} \quad \Rightarrow \quad v = \sqrt{\frac{e^2}{4\pi\epsilon_0 me r}} = \frac{e}{2\sqrt{\pi\epsilon_0 me r}} \\ &= \frac{e^2}{4(\pi r)^{3/2}\sqrt{\epsilon_0 me}} \quad = 2.188 \cdot 10^6 \frac{m}{s} \\ \Rightarrow \mu &= \frac{e^2}{4} \sqrt{\frac{r}{\pi\epsilon_0 me}} = (1.602 \cdot 10^{-19} C)^2 \frac{1}{4} \sqrt{\frac{5.29 \cdot 10^{-10} m}{\pi(8.85 \cdot 10^{-12} \frac{C^2}{Nm^2})(9.11 \cdot 10^{-31} kg)}} \\ &= 9.27 \cdot 10^{-24} A \cdot m^2 \end{aligned}$$

b. Find the torque on an electron orbiting in the xy-plane in a uniform magnetic field in positive x direction. (2 points)

counterclockwise



$$\begin{aligned} \vec{\tau} &= \vec{\mu} \times \vec{B} = |\vec{\mu}| \hat{k} \times |\vec{B}| \hat{i} \\ &= |\vec{\mu}| |\vec{B}| \hat{j} \\ &= (18.6 \cdot 10^{-26} Nm) \hat{j} \end{aligned}$$

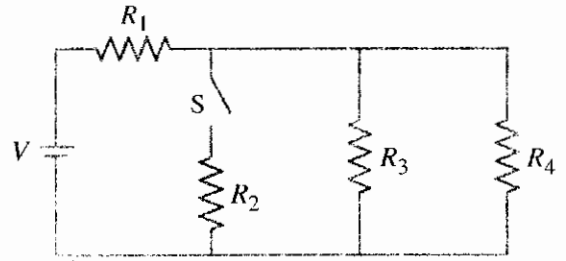
of 2 mT

c. How, if at all, will the H-atom respond, i.e. how will it move/turn?

- Proton at rest doesn't feel magnetic force.
- Electron orbit feels torque: rotating around \hat{j} , anti-clockwise

3. Equivalent Resistance

Consider the circuit on the right with $R_1=R_2=R_3=R_4=5k\Omega$ and $V=24V$.



a. What is the equivalent resistance of the circuit before and after the switch S has been closed? (2 points)

$$R_{eq} = R_1 + R_{eq}' \Rightarrow R_{eq} = \begin{cases} 7.5 k\Omega; & \text{before} \\ 6.6 k\Omega; & \text{after} \end{cases}$$

$$\text{Before: } R_{eq}' = \left(\frac{1}{R_3} + \frac{1}{R_4} \right)^{-1} = \frac{1}{2} \cdot 5k\Omega$$

$$\text{After: } R_{eq}' = \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right)^{-1} = \frac{1}{3} \cdot 5k\Omega$$

b. What is the current through R_1 before the switch is closed? (2 points)

$$I = \frac{V}{R_{eq}} = \frac{24V}{7.5 \cdot 10^3 \Omega} = \underline{\underline{3.2 \text{ mA}}}$$

c. How much power is the circuit dissipating when the switch is closed? (2 points)

$$P = IV = V^2 / R_{eq, \text{after}} = \underline{\underline{86.4 \text{ mW}}}$$

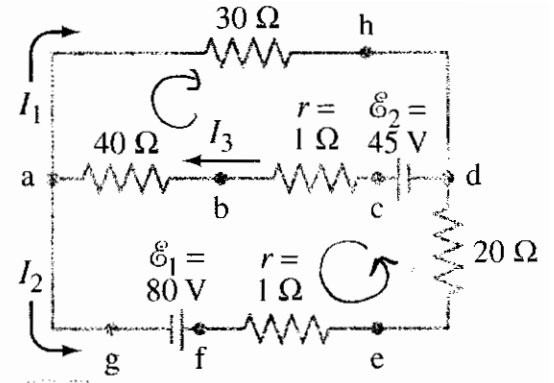
d. What is the voltage across R_4 when the switch is open? (2 points)

$$\text{From b: } I = 3.2 \text{ mA through } R_1 \Rightarrow V_1 = R_1 I_1 = 16V$$

$$\Rightarrow V_4 = 24V - 16V = \underline{\underline{8V}}$$

5. **Kirchhoff's Laws**

Consider the circuit on the right, and calculate the currents I_1 , I_2 , and I_3 . (3 points)



Junction: $I_3 = I_1 + I_2 = \underline{1.72 A}$

Loop 1 (counterclockwise):

$$\mathcal{E}_1 - I_2 r - 20 \Omega I_2 + \mathcal{E}_2 - r I_3 - 40 \Omega I_3 = 0$$

Loop 2 (clockwise): $-30 \Omega I_1 + \mathcal{E}_2 - r I_3 - 40 \Omega I_3 = 0$

$$\begin{cases} \mathcal{E}_1 + \mathcal{E}_2 - 21 \Omega I_2 - 41 \Omega (I_1 + I_2) = 0 \\ \mathcal{E}_2 - 30 \Omega I_1 - (I_1 + I_2) 41 \Omega = 0 \end{cases}$$

$$I_1 = \frac{1}{71 \Omega} (\mathcal{E}_2 - 41 \Omega I_2) = \underline{-0.86 A}$$

$$\mathcal{E}_1 + \mathcal{E}_2 - 21 \Omega I_2 - \frac{41}{71} (\mathcal{E}_2 - 41 \Omega I_2) = 0$$

$$I_2 = \frac{\mathcal{E}_1 + \frac{30}{71} \mathcal{E}_2}{62 \Omega - \frac{(41 \Omega)^2}{62 \Omega \cdot 71 \Omega}} = \underline{2.58 A}$$

6. **Concept Question**

You are given a copper bar of dimensions 3 cm × 5 cm × 8 cm and asked to attach leads to it in order to make a resistor. If you want to achieve the smallest possible resistance, you should attach the leads to the opposite faces that measure

a. 3 cm × 5 cm.

b. 3 cm × 8 cm.

c. 5 cm × 8 cm. *want maximal cross section.*

d. Any pair of faces produces the same resistance.

