Physics 172 **PRINCIPLES OF PHYSICS II** **WQ 2009**

First Midterm Exam

February 2, 2009

Name ___Key___

INSTRUCTIONS: Complete all problems. Show all work in the space provided. No books or notes allowed. Partial credit will be given for problems correctly set up even if incorrectly answered. No credit will be given for a numerical answer without supporting work. One point will be deducted from each answer without correct units. The time limit for this examination is 1 hour and 20 minutes. **Good luck!**

Constants

$\mu_0 = 4\pi x 10^{-7}$ Tm/A $\quad m_p = 1.673x10^{-27}$ kg $\quad$ 1 eV = $1.602x10^{-19}$ J

$\varepsilon_0 = 8.854x10^{-12}\ C^2/Nm^2$ $\quad m_e = 9.110x10^{-31}$ kg $\quad$ e = $1.602x10^{-19}$ C

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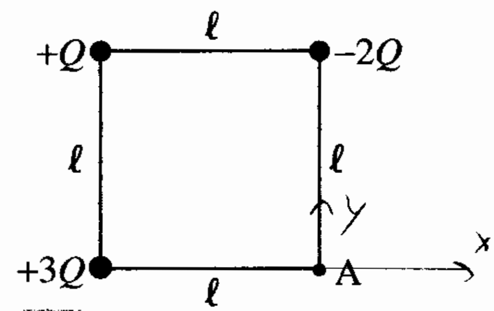
1. **Adding Electric Fields**



Consider three charges and a point A forming a square, see sketch below. When Q = 4nC, and the square is 10 cm wide, calculate the net electric field vector (magnitude and direction **or** x & y components in your choice of a coordinate system) at point A.

(3 points)

Field at A will be at $-45°$ from $+Q$, at $+90°$ from $-2Q$, and at $0°$ from $+3Q$.

Strengths: $E_Q = \frac{kQ}{r^2} = \frac{kQ}{(\sqrt{2}\ell)^2} = \frac{1}{2} k \frac{Q}{\ell} = (4.5\cdot 10^9)\left(\frac{4\cdot 10^{-9}}{0.1^2}\right) \frac{N}{C} = 1800 \frac{N}{C}$

$E_{-2Q} = 4 E_Q = 7200 \frac{N}{C}$

$E_{3Q} = 6 E_Q = 10800 \frac{N}{C}$

$$\vec{E}_{net} = E_Q \begin{pmatrix} \cos -45° \\ \sin -45° \end{pmatrix} + E_{-2Q} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + E_{3Q} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= E_Q \begin{pmatrix} 1/\sqrt{2} + 6 \\ -1/\sqrt{2} + 4 \end{pmatrix} \simeq \begin{pmatrix} +12100 \frac{N}{C} \\ +5900 \frac{N}{C} \end{pmatrix}$$

2. **Gauss's Law**

A non-conducting sphere of radius Rout has a spherical cavity of radius R_{in} centered at the sphere's center. A charge Q=10mC is distributed uniformly over the volume of the "shell", $R_{in}<r<R_{out}$, with $2R_{in}=R_{out}=10$cm.

a. What is the electric flux through the outer surface of the shell? (2points)

Electric field is radially outward, parallel to surface vector everywhere.
Surface of sphere of radius R_{out} : $4\pi R_{out}^2$. Electric field strength:

$$E = k\frac{Q}{R_{out}^2} = (9\cdot 10^9)\frac{10^{-2}C}{(0.1m)^2}$$

$$\Rightarrow \Phi_E = E\cdot A = k\frac{Q}{R_{out}^2}\cdot 4\pi R_{out}^2 = 4\pi k Q = 1.13\cdot 10^9 \frac{N}{C}m^2$$

b. Find the electric field at the center of the cavity. (2 points)

There are no charges inside the cavity. Gauß' law: $\oint \vec{E}\cdot d\vec{A} = 0$

$$\Rightarrow \vec{E} = 0$$

c. Calculate the electric field 8cm away from the center of the sphere. (2 points)

This is inside of the charged shell, so choose gaussian surface with r = 8cm. $\oint \vec{E}\cdot d\vec{A} = E\int dA = E\,4\pi r^2$

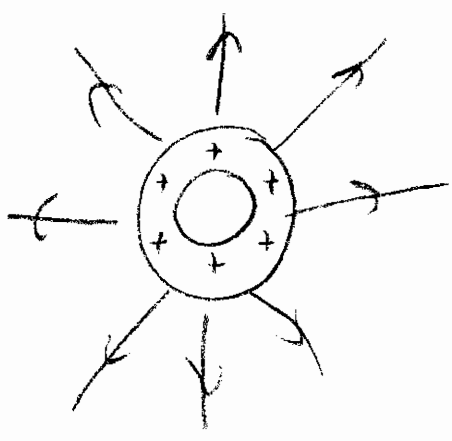
Enclosed charge: $Q_{encl} = Q\frac{\frac{4\pi}{3}(r^3-R_{in}^3)}{\frac{4\pi}{3}(R_{out}^3-R_{in}^3)} = \frac{(r^3-R_{in}^3)}{R_{out}^3-R_{in}^3}Q$ $= \frac{Q_{encl}}{\varepsilon_0} \Rightarrow E = \frac{Q}{4\pi\varepsilon_0}\frac{r-\frac{R_{in}^3}{r^2}}{R_{out}^3-R_{in}^3}$

$$= kQ\frac{0.08m - \frac{(0.05m)^3}{(0.08m)^2}}{(0.1m)^3-(0.05m)^3}$$

$$= kQ\frac{6.05\cdot 10^{-2}m}{8.75\cdot 10^{-4}m^3}$$

$$= 6.22\cdot 10^9 \frac{N}{C}$$

d. Draw the electric field outside of the shell, i.e. for r > R_{out}. (2 points)



Radially outward; same as point charge with +Q.

3. **Potential**

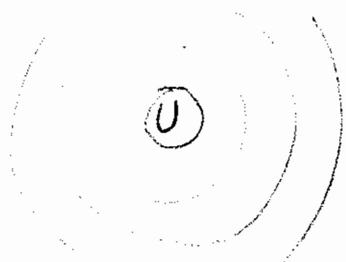
A proton (mass see constants) is shot with a speed of 2000m/s towards a Uranium nucleus (charge: +92 e) at rest from a distance of 2mm. Approximate the nucleus as a uniformly charged, infinitely heavy sphere. of tiny radius.

a. What is the potential ~~of~~ due to the nucleus at the initial position of the proton, if we set the potential to zero at infinite distance? (2 points)

$$V = k\frac{Q}{r} = \left(9\cdot 10^{9}\,\frac{Nm^2}{C^2}\right)\frac{92 \cdot 1.602\cdot 10^{-19}C}{2\cdot 10^{-3}m}$$

$$= 6.6\cdot 10^{-5}\,V$$

b. Draw a set of equipotential surfaces for the Uranium nucleus. (2 points)



Equipotential surfaces are concentric spheres.

c. How far out is the 10V equipotential surface? (2 points)

$$V \overset{!}{=} 10V = k\frac{Q}{r} \Rightarrow r = \frac{kQ}{10V} = \frac{\left(9\cdot 10^{9}\frac{Nm^2}{C^2}\right)\left(92\cdot 1.602\cdot 10^{-19}C\right)}{10\ J/C}$$

$$= 1.3\cdot 10^{-8}\,m$$

d. How close will the proton get to the center of the Uranium nucleus? (2 points)

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f \quad ; \; K_f = 0 \text{ since } V_f = 0$$

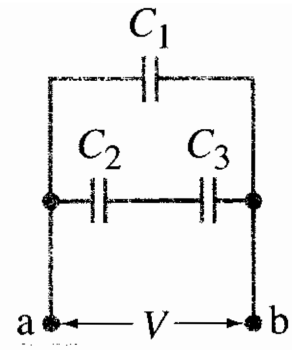
$$\frac{1}{2}m_p v_i^2 + e\,V_{(2mm)} = e\,\frac{kQ}{r_f}$$

$$\Rightarrow r_f = \frac{ekQ}{\frac{1}{2}m_p v_i^2 + e\cdot 6.6\cdot 10^{-5}V} = \frac{2.12\times 10^{-26}\,Nm^2}{3.3\cdot 10^{-21}Nm + 1.06\cdot 10^{-23}Nm}$$

$$\approx 6.4\cdot 10^{-6}\,m$$

4. **Capacitors**

Consider the circuit on the right.



a. Determine the equivalent capacitance of the circuit for $C_1 = C_2 = C_3 = 30\text{pF}$. (2 points)

C_2 & C_3 in series : $C'_{eq} = \frac{1}{1/C_2 + 1/C_3}$

C_1 parallel to C_2 & C_3 : $C_{eq} = C_1 + C'_{eq} = C_1 + \frac{C_2 C_3}{C_2 + C_3}$

$= 30\text{pF} + \frac{900\text{pF}}{60} = 45\text{pF}$

b. Find the voltages across each of the three capacitors if the potential difference between a and b is 12V. (2 points)

Voltage across C_1 is the full 12V,

Voltage across C_2 & C_3 combined is 12V, since $C_2 = C_3$, $V_2 = V_3$, so

$V_2 = V_3 = 6V$

c. How much charge is stored on each capacitor for V=12V? (*Hint: the charge on the series combination is the same as the charge on each of the individual capacitors.*) (2 points)

$Q = CV \Rightarrow Q_1 = C_1 V_1 = (30\text{pF})(12V) = 3.6 \cdot 10^{-10} C$

$Q_2 = Q_3 = C_2 V_2 = C_3 V_3 = (30\text{pF})(6V) = 1.8 \cdot 10^{-10} C$

d. Determine the total energy stored in the circuit for V=12V. (2 points)

$U = \frac{1}{2} C_{eq} V^2 = \frac{1}{2}(45\text{pF})(12V)^2$

$= 3.24 \cdot 10^{-9} J$

5. **Concept Question**

An electron sits in the constant electric field of two large, conducting, charged plates. It is close to the negatively charged plate. Which of the following statements is correct? (Circle the right answer.) (1 point)

a. The electric potential at the electron's place is high compared to the potential close to the positively charged plate.

b. The electron's potential energy is high compared to the potential energy it would have sitting close to the positively charged plate.

c. The work done by an external force on the electron as it moves from the negative to the positive plate is positive. $\Delta U = U_f - U_i$ is neg. $\Rightarrow W_{ext} = \Delta U$ is neg.

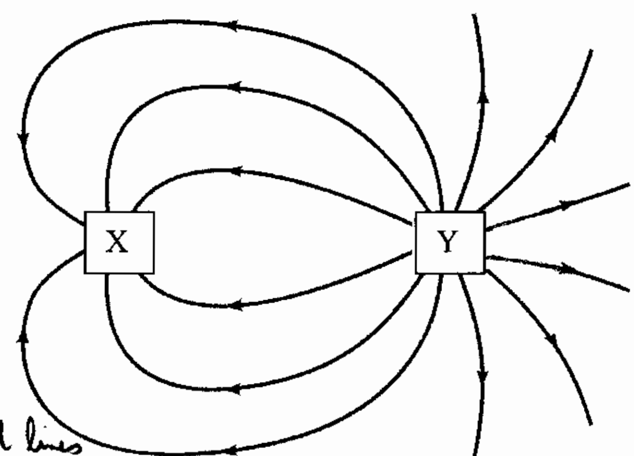
d. The work done by the electric field on the electron as it moves from the negative to the positive plate is negative. $\Delta U = -W_{int}$

e. As the electron moves from the negative to the positive plate, it travels on an equipotential line.

6. **Concept Question**

Consider the field lines to the right. Which of the following statements is correct? (Circle the right answer.)

(1 point)



a. The charge at X is positive, the one at Y is negative. No, field lines run towards X.

b. The amount of charge at X is larger than the one at Y. No, more field lines at Y

c. The charges are equal. No, different density of field lines

d. The charge at X is negative and its amount smaller than the amount of charge a Y.

e. The charge at X is a point charge, at Y there is a dipole with axis perpendicular to the line connecting the points X and Y. No. Dipole has more complicated field.