

Second Midterm Exam

February 24, 2010

Name Key

INSTRUCTIONS: Complete all problems. Show all work in the space provided. No books or notes allowed. Partial credit will be given for problems correctly set up even if incorrectly answered. No credit will be given for a numerical answer without supporting work. One point will be deducted from each answer without correct units. The time limit for this examination is 1 hour and 20 minutes. **Good luck!**

Constants

$$\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

$$m_p = 1.673 \times 10^{-27} \text{ kg}$$

$$m_e = 9.110 \times 10^{-31} \text{ kg}$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

1. RC circuit

A capacitor of capacitance 30pF holding a charge of 5 nC is being discharged via a 2 k Ω resistor.

- a. What is the "half-life" of the charge on the capacitor, i.e. how long will it take for the capacitor to lose half its charge? (Hint: Use $Q=CV$.) (2 points)

$$Q(t) = CV_c(t) = \underbrace{CV_0}_{Q_0} e^{-t/RC} \Rightarrow \frac{Q(t)}{Q_0} = e^{-t/RC} = \frac{1}{2}$$

$$\Rightarrow t = RC \ln 2 = (60 \cdot 10^{-12} \cdot 10^3 \ln 2) \text{ s} = \underline{\underline{4.16 \cdot 10^{-8} \text{ s}}}$$

- b. What is the voltage across the capacitor at this point in time? (2 points)

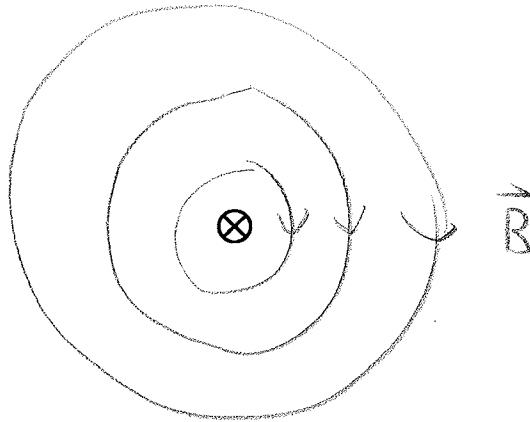
$$V_c = \frac{1}{2} V_0 = \frac{1}{2} \frac{Q_0}{C} = \frac{1}{2} \frac{5 \text{ nC}}{30 \text{ pF}} = \underline{\underline{83.3 \text{ V}}}$$

- c. What is the maximal current that will flow? (Hint: Use Ohm's law.) (2 points)

$$I_{\max} = \frac{V_{\max}}{R} = \frac{Q_0}{RC} = \frac{5 \text{ nC}}{(2000 \Omega)(30 \cdot 10^{-12} \text{ F})} = 0.083 \text{ A} = \underline{\underline{83 \text{ mA}}}$$

2. Ampere's Law

The cross section of a long straight wire of radius R is shown in the diagram. A current I flows through the wire. The current is directed into the paper.



- a) Use Ampere's Law to derive an expression for the magnitude of the magnetic field at a distance r from the center of the wire for values of r outside the wire ($r > R$). Carefully describe the reasoning you use in each step of the derivation. (Hint: Compare to actual result on formula sheet.) (2 points)

Ampere's law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$

For $r > R$ all current is enclosed: $I_{\text{enc}} = I$.

The magnetic field is circular around the wire \rightarrow choose an Amperian loop of circular shape with radius $r > R$. Then $\vec{B} \parallel d\vec{l}$, so $\vec{B} \cdot d\vec{l} = B dl \cos 0^\circ = B dl$. Also, since $r = \text{const}$,

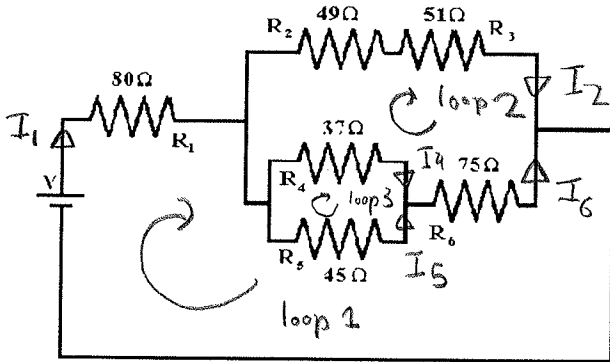
$$B(r) = \text{const.} \Rightarrow \oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = B(2\pi r)$$

$$\text{So } B(2\pi r) = \mu_0 I \quad \text{or} \quad \underline{B = \frac{\mu_0 I}{2\pi r}} \quad \checkmark$$

- b) Use the right hand rule to determine the direction of the magnetic field (describe how you use the rule). Draw a magnetic field line on the diagram and indicate the field direction. (2 points)

Thumbs in direction of current, fingers of right hand will point in direction of magnetic field \rightarrow clockwise.

3. Equivalent Resistance & Kirchhoff's rules: Consider the circuit below with $V=12V$.



a. What is the current through R_1 ? (Hint: Use the equivalent resistance of the circuit.) (2 points)

$$I = \frac{V}{R_{eq}} \Rightarrow R_{eq} = R_1 + \frac{1}{\frac{1}{R_2 + R_3} + \frac{1}{R_6 + \frac{1}{\frac{1}{R_4} + \frac{1}{R_5}}}} = 128.8 \Omega$$

$$\Rightarrow I = \frac{12V}{128.8 \Omega} = 0.093 A = \underline{\underline{93 mA}}$$

b. Set up a set of Kirchhoff's rules, specifying how many (and which) unknown quantities exist. Be sure to label all currents and indicate their directions. (2 points)

All currents marked above: I_1, I_2, I_4, I_5, I_6 are unknown.

Need 5 eqns:

3 loop rules:

$$V - I_1 R_1 - I_5 R_5 - I_6 R_6 = 0$$

$$-I_4 R_4 + I_5 R_5 = 0$$

$$-I_2 (R_2 + R_3) + I_6 R_6 + I_4 R_4 = 0$$

2 junction rules

$$I_2 + I_6 = I_1$$

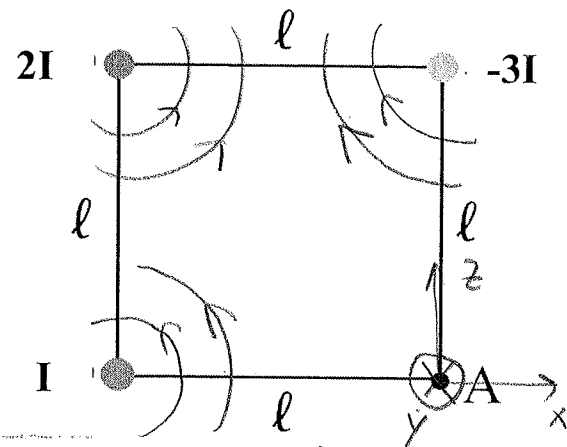
$$I_4 + I_5 = I_6$$

c. How much power does the circuit use? (2 points)

$$P = VI = (12V)(0.093A) = \underline{\underline{1.116 W}}$$

4. Particle in a magnetic field

An electron at position A sits in the vicinity of three wires carrying currents I , $2I$ and $-3I$, see sketch. The positive currents are coming out of the page ($-y$ direction), the negative one is going into the page. The electron moves up ($+z$ direction) with velocity 4000 m/s. Let $I = 10$ A and $l = 5 \mu\text{m}$.



- a. Calculate the strength of the magnetic field of each of the wires at the position of the electron. (2 points)

$$B_I = \frac{\mu_0}{2\pi} \frac{I}{l} = 0.4 \text{ T}$$

$$B_{2I} = \frac{\mu_0}{2\pi} \frac{2I}{\sqrt{2}l} = \sqrt{2} B_I = 0.566 \text{ T}$$

$$B_{3I} = \frac{\mu_0}{2\pi} \frac{3I}{l} = 3B_I = 1.2 \text{ T}$$

- b. Determine the direction of each of the three magnetic fields at point A. (2 points)

All fields are circular around their wires; B_I & B_{2I} counterclockwise.

At point A: \vec{B}_I : up ($+z$) \vec{B}_{3I} : left ($-x$)

\vec{B}_{2I} : 45° from x -axis towards z -axis

- c. Calculate the total magnetic field (strength & direction or cartesian coord.) at point A. (2 points)

$$\begin{aligned} \vec{B}_{\text{total}} &= \vec{B}_I + \vec{B}_{2I} + \vec{B}_{3I} \\ &= B_I \left(\hat{k} + \sqrt{2} \frac{1}{\sqrt{2}} (\hat{i} + \hat{k}) + 3(-\hat{i}) \right) \\ &= 2B_I (\hat{k} - \hat{i}) = \begin{pmatrix} -0.8 \text{ T} \\ 0 \\ 0.8 \text{ T} \end{pmatrix} \end{aligned}$$

- d. What is the force (strength & direction or cartesian coord.) the on the electron due to the (total) magnetic field? (2 points)

$$\begin{aligned} \vec{F} &= q \vec{v} \times \vec{B} = -e (4000 \frac{\text{m}}{\text{s}}) (0.8 \text{ T}) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \\ &= -3200e \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \\ &= 5.126 \cdot 10^{-16} \text{ N } \hat{j} ; \text{ positive } y\text{-direction} \end{aligned}$$

5. Concept Question

Rank the magnetic fields produced at point P by two currents of equal strengths starting with the largest upward-pointing field. Let \otimes symbolize a current going into the page and \oplus a current coming out of the page). Count a magnetic field pointing up as positive.



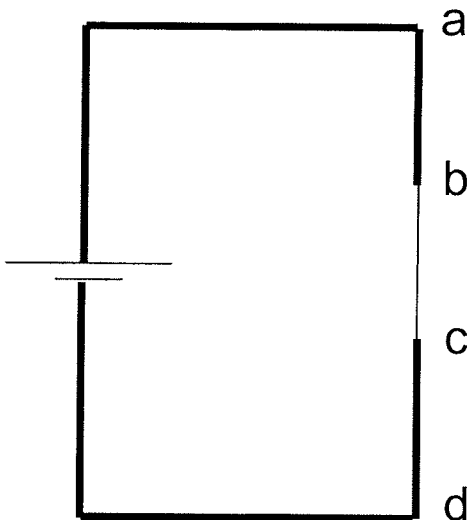
(Circle the correct answer)

(1 point)

BCAD

- a. ABCD
- b. BADC
- c. CDAB
- d. ADBC
- e. DCAB
- f. None of the above

6. Resistance and Resistivity



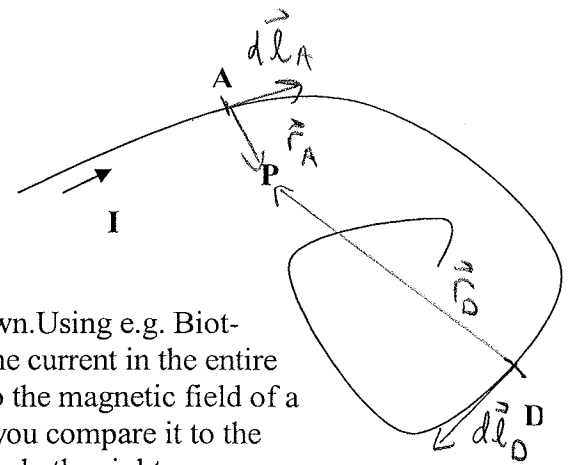
A wire connected to a battery has sections of different diameters. How do the **potential differences** V_{ab} , V_{bc} and V_{cd} compare? The distances between a,b,c,d are equal. Circle the right answer. (1 point)

- a. $V_{ab} > V_{bc} > V_{cd}$ *Same current through*
- b. $V_{ab} = V_{bc} > V_{cd}$ *all wires, but*
- c. $V_{ab} = V_{bc} < V_{cd}$ $R_{ab} < R_{bc}$
- d. $V_{ab} = V_{bc} = V_{cd}$ $R_{ab} = R_{cd}$
- e. None of the above.

$\Rightarrow V_{ab} = IR_{ab} = V_{cd} < V_{bc}$

7. Biot-Savart's Law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$



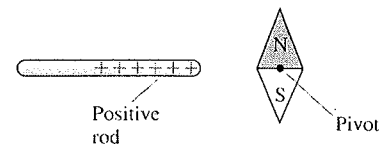
A current I is flowing in an irregularly shaped wire as shown. Using e.g. Biot-Savart's law, we can figure out the magnetic field due to the current in the entire wire at point P. What can you say about the contribution to the magnetic field of a tiny bit of wire located roughly at position A will make if you compare it to the contribution of a tiny bit of wire roughly at position D? Circle the right answer.

(1 point)

- a. The contribution from point D will be larger. *farther away ↓*
- b. The contribution from point D will point in the direction of the wire at D. *↓ $d\vec{B} \perp d\vec{l}$*
- c. The contribution from point A will be pointing somewhere in the plane of the paper. *↓ $d\vec{B} \perp d\vec{l}$*
- d. The contributions from point A and point D will point in the same direction. ✓
- e. The contribution from point A will point out of the page. *into page ↓*
- f. None of the above.

8. Concept Question

What happens when a charged object is brought near a magnet? Circle the right answer. (1 point)



- a. The south pole goes toward the positive
- b. The north pole rotates toward the positive
- c. None of the above.

Only moving electric charges produce a magnetic field.

9. Right Hand Rule

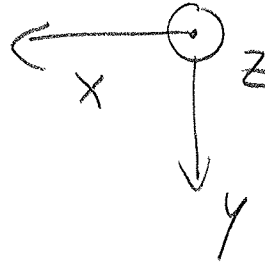
Let the x-axis point to the left and the z-axis point out of the page. Where does the y-axis point to? Circle the right answer. (1 point)

a. To the right.

b. Into the page.

c. Up

d. None of the above.



Use right hand rule

x - thumb

y - index finger

z - middle finger