Physics 172 **PRINCIPLES OF PHYSICS II** **WQ 2010**

First Midterm Exam

February 1, 2010

Name Key

INSTRUCTIONS: Complete all problems. Show all work in the space provided. No books or notes allowed. Partial credit will be given for problems correctly set up even if incorrectly answered. No credit will be given for a numerical answer without supporting work. One point will be deducted from each answer without correct units. The time limit for this examination is 1 hour and 20 minutes. **Good luck!**

Constants

$\mu_0 = 4\pi x10^{-7}$ Tm/A $m_p = 1.673x10^{-27}$ kg 1 eV = $1.602x10^{-19}$ J
$\varepsilon_0 = 8.854x10^{-12}$ C^2/Nm^2 $m_e = 9.110x10^{-31}$ kg e = $1.602x10^{-19}$ C

Area & Surface Formulae

Surface area of disk of radius R: πR^2
Surface area of cylinder mantle of radius R and length l: $2\pi Rl$
Surface area of sphere of radius R: $4\pi R^2$

Volume of cylinder of radius R and length l: $\pi R^2 l$
Volume of sphere of radius R: $4\pi R^3/3$

1. **Gauss's Law**

A long, **conducting** cylindrical pipe has an inner radius R_{in}, an outer radius R_{out}, and a length L. An amount of charge +Q is put on it by touching it briefly with a charged object.

a. What is the electric field (magnitude and direction) inside of the pipe, i.e. for $r < R_{in}$? (2 points)

No charge inside, $\frac{Q_{encl}}{\varepsilon_0} = 0 = \oint \vec{E} \cdot d\vec{A} = E(2\pi R L)$

$\Rightarrow E = 0$

b. What is the electric field (magnitude and direction) in the body of the pipe ($R_{in} < r < R_{out}$)? (2 points)

Body is a conductor: $E = 0$

c. What is the electric flux through the outer surface of the cylinder? (2 points)

$$\oint \vec{E} \cdot d\vec{A} = \Phi_E = \frac{Q_{encl}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$; all charge enclosed

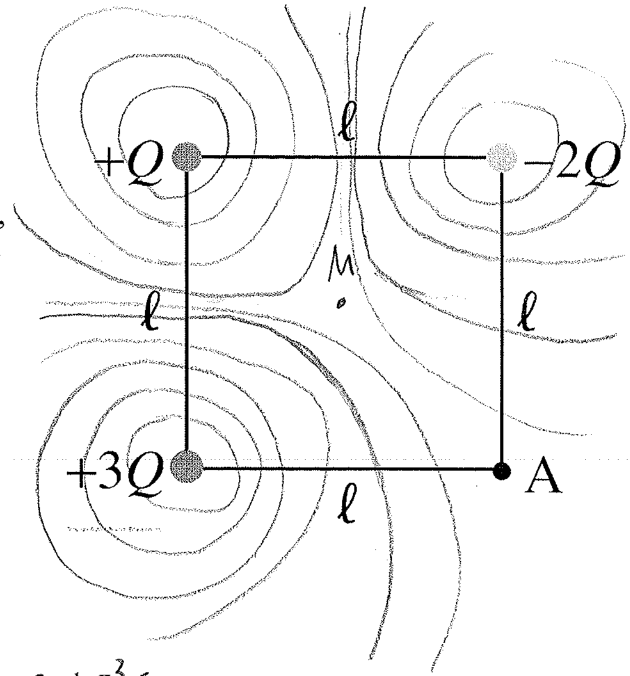
d. What is the electric field (magnitude and direction) outside the pipe ($r > R_{out}$)? (2 points)

$$\oint \vec{E} \cdot d\vec{A} = E \int dA = E(2\pi r L) = \frac{Q_{encl}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E(r) = \frac{Q/L}{2\pi\varepsilon_0} \frac{1}{r}$$

Direction: away from the axis of the cylinder.

2. **Potential**



Consider three charges and a point A forming a square, see sketch at right with Q = 3mC, and l = 5 cm. Set the electric potential to zero at infinite distance.

a. What is the electric potential at point A? (2 points)

$$V(A) = V_{3Q}(A) + V_{Q}(A) + V_{-2Q}(A)$$

$$= k\left(\frac{3Q}{\ell} + \frac{Q}{\sqrt{2}\ell} + \frac{-2Q}{\ell}\right)$$

$$= \frac{kQ}{\ell}\left(1+\frac{1}{\sqrt{2}}\right) = \left(9\cdot 10^9 \frac{Nm^2}{C^2}\right)\frac{3\cdot 10^{-3}C}{5\cdot 10^{-2}m}\left(1+\frac{1}{\sqrt{2}}\right) = \underline{\underline{9.22\cdot 10^8\ V}}$$

b. What is the potential in the middle of the square? (2 points)

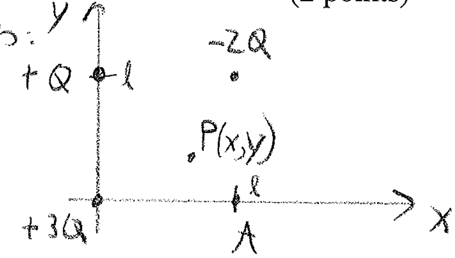
$$V(M) = \frac{k}{\ell}\frac{2}{\sqrt{2}}(3Q+Q-2Q) = \frac{2\sqrt{2}}{\ell}kQ$$

$$= \underline{\underline{15.3\cdot 10^8\ V}}$$

c. Draw a set of equipotential lines into the diagram. (2 points)

d. Which points other than the ones at infinity, if any, are on zero potential? Derive an equation decribing such points in the plane of the charges or explain why none exist (2 points)

Need $V \equiv 0$. Choose coordinates:



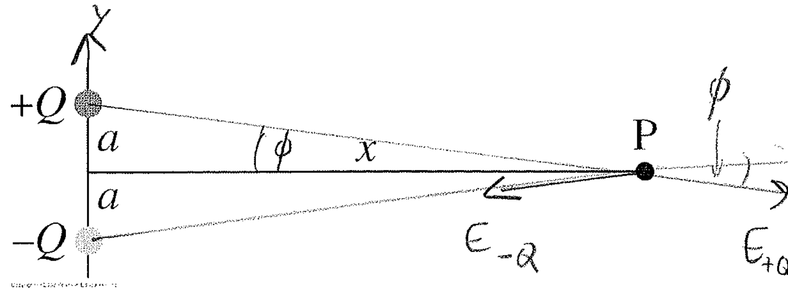
Then a point $P(x,y)$ is $\sqrt{x^2+y^2}$ away from $+3Q$, $\sqrt{x^2+(\ell-y)^2}$ from Q, and $\sqrt{(\ell-x)^2+(\ell-y)^2}$ from $-2Q$.

So $$V(x,y) = kQ\left(\frac{3}{\sqrt{x^2+y^2}} + \frac{1}{\sqrt{x^2+(\ell-y)^2}} - \frac{2}{\sqrt{(\ell-x)^2+(\ell-y)^2}}\right)$$

We have $V(x,y)=0$ when the term in the brackets vanishes, which is the condition sought.

3. Electric Field of a Dipole

Consider a point P equally distant from two opposite charges as in the sketch on the right.



a. Determine the direction and magnitude (or Cartesian components) of the electric field at point P if the two charges are separated by a distance $2a$. Express your answer in terms of Q, x, a, and k (or ε_0). (2 points)

The electric fields are radially outward from the charges (inward for neg. charge), so the x-components will cancel on the x-axis for points, and the y-components are the same, so

$$\vec{E}_{total} = \vec{E}_{Q} + \vec{E}_{-Q}\ , \quad E_{total,y} = E_{Q,y} + E_{-Q,y}$$

$$= k\frac{Q\sin(\phi)}{\sqrt{a^2+x^2}^2}(1+1)$$

$$\sin\phi = \frac{a}{\sqrt{a^2+x^2}}$$

$$= \frac{-2kQ}{\sqrt{a^2+x^2}^3}$$

b. What is the **strength** and **direction** of the force that the dipole would exert on an electron sitting at P if a = 2nm, Q = 1.5 nC, and x = 3mm? (2 points)

$$\vec{F} = q\vec{E} = -e\vec{E}\ , \quad |\vec{F}| = eE = \frac{2kQe}{\sqrt{a^2+x^2}^3} \approx \frac{2kQe}{x^{3/2}}$$

$\vec{E}$ points in negative y direction, so $\vec{F}$ points in positive y direction

$a \ll x$

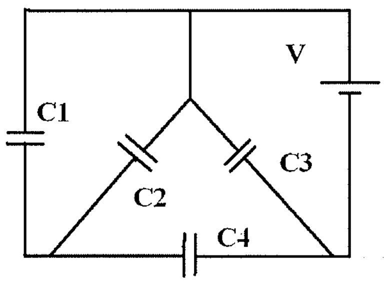
$$= 2.63\cdot 10^{-14}\,N$$

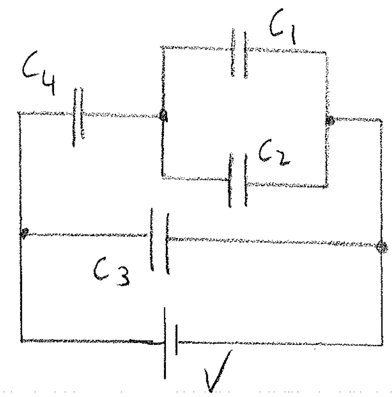
c. What is the potential at the point P? (1 point)

$V = 0$ since P is equidistant from opposite charges.

$$= \frac{kQ}{R} + \frac{-kQ}{R}\,.$$

4. **Equivalent Capacitance**





Consider the circuit above with V=12V, C1=C3=10mF and C2=C4=20mF.

a. Redraw the circuit such that its capacitors are explicitly either in series or parallel. (2 points)

b. What is the equivalent capacitance of the circuit? You can do this by using the result of a. or by writing potential differences for various pathways through the network in terms of the charges on the capacitors, and the capacitances. (2 points)

C_1 & C_2 parallel: $C_{eq} = C_1 + C_2$

C_3 parallel to C_4 in series with C_{eq}:

$$C_{eq}' = C_3 + \frac{1}{1/C_4 + 1/C_{eq}} = C_3 + \frac{C_4(C_1+C_2)}{C_1+C_2+C_4}$$
$$= 10\,mF + \frac{20(10+20)}{50}\,mF$$
$$= \underline{22\,mF}$$

c. What is the charge on the capacitor C3? You can figure this out without using the previous results. (2 points)

We have the full voltage across C_3, so $Q_3 = C_3 V = (10mF)(12V)$
$= \underline{120\,mC}$

d. What is the energy stored in C3? (1 point)

$$U_3 = \frac{1}{2} Q_3^2/C_3 = \underline{0.72\,J}$$

e. Let C3 be a capacitor with square plates a distance 1mm apart. Find the width of its plates. (2 points)

$$C_3 = 10\,mF = \varepsilon_0 \frac{A}{d} = \frac{\varepsilon_0}{d} l^2 \Rightarrow l = \sqrt{\frac{d C_3}{\varepsilon_0}}$$
$$= \underline{1063\,m}$$

Pretty big...

5. Concept Question

Suppose a conductor carries a net charge +Q and contains a cavity, inside of which resides a point charge +q. Which of the following statements is true? (Circle the right answer.)
(1 point)

a. The electric field inside the conductor is larger due to the additional positive point charge. No electric field within a conductor.

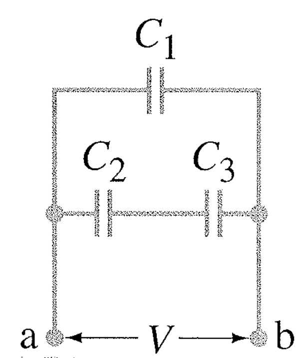
b. All the positive individual charges of the conductor will be spread out uniformly within the conductor. No charges within a conductor.

c. The outside surface of the conductor will carry a charge larger than Q.

d. The inside surface of the conductor (the area around the cavity) will carry an induced charge of –Q. No, of $-q$.

6. Concept Question

Consider the capacitor circuit on the right. While the voltage is held constant, someone inserts a dielectric into the capacitor C_1. How does the circuit react? (Circle the right answer.) (1 point)



a. The voltage across C_1 increases. No, $V=$ const.

b. The voltage across C_3 increases. No, $V=$ const. and C_2, C_3 const. so const. charges.

c. The charge on C_2 increases. No, no change of C_2, C_3, so no change in charges.

d. The charge on C_1 increases. Yes, $V=$ const. and C_1 goes up, so $Q_1 = C_1 V$ up.

e. Nothing changes.

f. None of the above.