

Final Exam

March 18, 2009

Name Key

INSTRUCTIONS: Complete all problems. Show all work in the space provided. No books or notes allowed except for the three "official" formula sheets. Partial credit will be given for problems correctly set up even if incorrectly answered. No credit will be given for a numerical answer without supporting work. One point will be deducted from each answer without correct units. The time limit for this examination is 2 hours. **Good luck!**

Constants

$$\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

$$m_{\text{proton}} = 1.673 \times 10^{-27} \text{ kg}$$

$$m_{\text{electron}} = 9.110 \times 10^{-31} \text{ kg}$$

$$k = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

1. Transformer

A transformer is operated with an in input voltage of 12V, having a number of primary windings of 300, and a number of secondary windings of 550. It delivers an output current of 400 mA.

- a) What is the output voltage? (2 points)

$$V_s = \frac{N_s}{N_p} V_p = \frac{550}{300} (12\text{V}) = \underline{\underline{22\text{V}}}$$

- b) What is the input current? (2 points)

$$I_p = \frac{N_s}{N_p} I_s = \frac{550}{300} (400\text{mA}) = \underline{\underline{733\text{mA}}}$$

- c) Is the transformer "step-up" or "step-down"? (1 point)

Voltage higher on output \rightarrow "step-up"

2. Induction

A circular loop of wire 10 cm in radius sits in the plane of the paper in a magnetic field coming out of the paper.

- a) If the magnetic field changes from +0.45 mT to -0.05 mT in 0.3 seconds, what is the induced emf? (2 points)

$$\begin{aligned}\mathcal{E} &= -N \frac{d\Phi_B}{dt} = -A \frac{dB}{dt} \approx -\pi r^2 \frac{\Delta B}{\Delta t} = -\pi (0.1 \text{ m})^2 \frac{-0.5 \text{ mT}}{0.3 \text{ s}} \\ &= \underline{\underline{0.0524 \text{ mV}}}\end{aligned}$$

- b) What is the direction of the induced current in this situation? (1 point)

Field gets weaker in \odot direction, need to produce \odot lines
 \Rightarrow counter-clockwise current

- c) If the loop changes its radius from 10 cm to 15 cm in 1.6 seconds when the magnetic field has a strength of 0.45 mT, what is the magnitude of the average induced emf? (2 points)

$$\begin{aligned}|\mathcal{E}| &= B \frac{\Delta A}{\Delta t} = B \pi \frac{\Delta r^2}{\Delta t} = B \pi \frac{r_2^2 - r_1^2}{1.6 \text{ s}} = B \pi \frac{(0.15 \text{ m})^2 - (0.1 \text{ m})^2}{1.6 \text{ s}} \\ &= \underline{\underline{0.011 \text{ mV}}}\end{aligned}$$

- d) What is the direction of the induced current in this situation? (1 point)

Adding \odot lines, need \otimes lines \Rightarrow clockwise

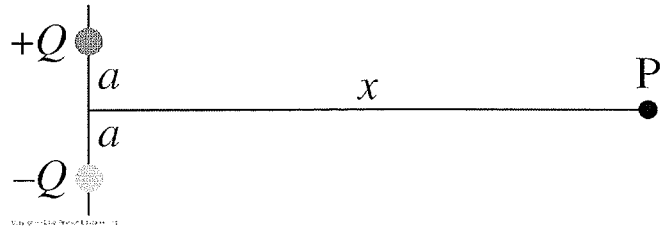
- e) If the radius of the loop is fixed at 15 cm, the magnetic field remains at 0.45 mT, and the loop is rotated (around an axis in the plane of the paper) at a frequency of 40 revolutions per second, what is the **amplitude** and the **frequency** of the emf? (2 points)

$$f = 40 \text{ Hz}$$

$$\begin{aligned}\mathcal{E}_0 &= NBA\omega = 1 (0.45 \text{ mT}) (\pi r^2) 2\pi (40 \text{ Hz}) \\ &= \underline{\underline{7.99 \text{ mV}}}\end{aligned}$$

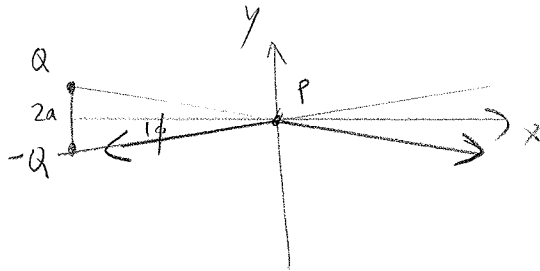
3. Electric Field of a Dipole

Consider a point P equally distant from two opposite charges as in the sketch on the right.



a. Determine the direction and magnitude of the electric field at point P if the two charges are separated by a distance $2a$. Express your answer in terms of Q , x , a , and k (or ϵ_0).

(2 points)



\Rightarrow X components cancel.

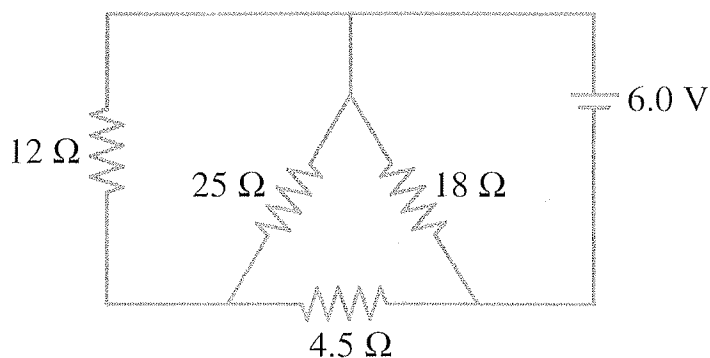
$$\begin{aligned}
 E_y &= E_{+y} + E_{-y} = 2k \frac{|Q|}{a^2 + x^2} (-\sin \phi) \\
 &= -2k \frac{|Q|}{a^2 + x^2} \frac{a}{\sqrt{a^2 + x^2}} \\
 &= -\frac{2k|Q|a}{(a^2 + x^2)^{3/2}}
 \end{aligned}$$

b. What is the **strength** and **direction** of the force that the dipole would exert on an electron sitting at P if $a = 2\text{nm}$, $Q = 1.5\text{ nC}$, and $x = 3\text{mm}$? (2 points)

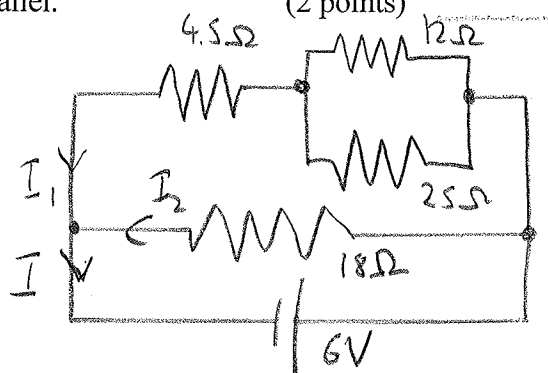
$$\begin{aligned}
 |F| &= qE = 2(1.602 \cdot 10^{-19}\text{ C})(8.99 \cdot 10^9 \frac{\text{N m}^2}{\text{C}^2}) \left[\frac{(1.5 \cdot 10^{-9}\text{ C})(2 \cdot 10^{-9}\text{ m})}{(4 \cdot 10^{-18}\text{ m}^2 + 9 \cdot 10^{-6}\text{ m}^2)^{3/2}} \right] \\
 &= \underline{\underline{3.2 \cdot 10^{-19}\text{ N}}}
 \end{aligned}$$

4. Equivalent Resistance

Consider the circuit on the right.



a. Redraw the circuit such that its resistors are explicitly either in series or parallel. (2 points)



b. What is the equivalent resistance of the circuit? (2 points)

$$R_{eq} = 4.5 \Omega + \frac{1}{\frac{1}{12 \Omega} + \frac{1}{25 \Omega}} = 12.6 \Omega$$

$$R_{eq} = \frac{1}{\frac{1}{18 \Omega} + \frac{1}{R_{eq}'}} = 7.41 \Omega$$

c. What is the current in the 18 Ohm resistor? (2 points)

$$V_{18} = 6V = IR \Rightarrow I = \frac{6V}{18 \Omega} = \underline{\underline{\frac{1}{3} A}}$$

d. What is the power dissipation in the 4.5 Ohm resistor? (2 points)

$$\text{Current through } 4.5 \Omega \text{ resistor: } V = 6V = I R_{eq}' \Rightarrow I = 0.476 A$$

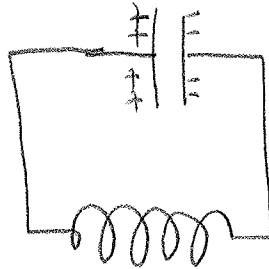
$$P = I^2 R = \underline{\underline{1.02 W}}$$

5. Qualitative Question

A capacitor is fully charged when it is connected to an inductor. All of the following are qualitative questions.

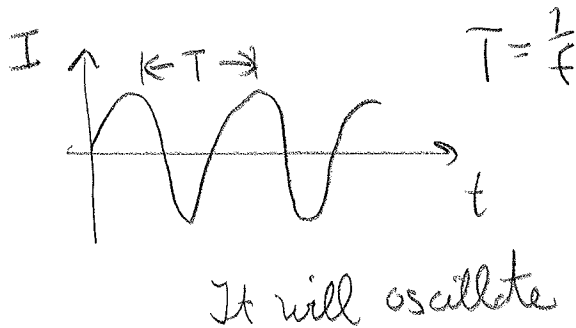
a. Sketch the circuit.

(2 points)



b. How will the current change in time?

(2 points)



c. Plot the charge on the capacitor's plates as a function of time.

(2 points)

