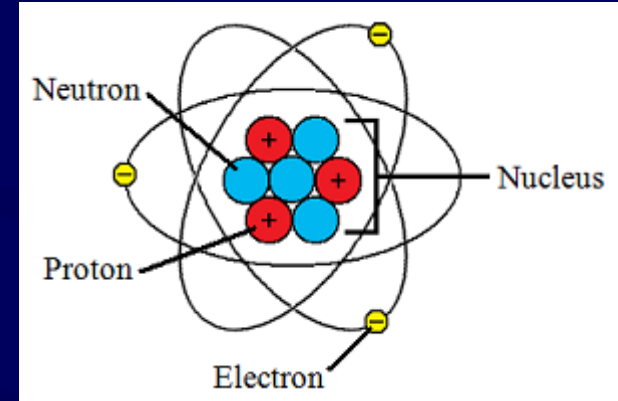


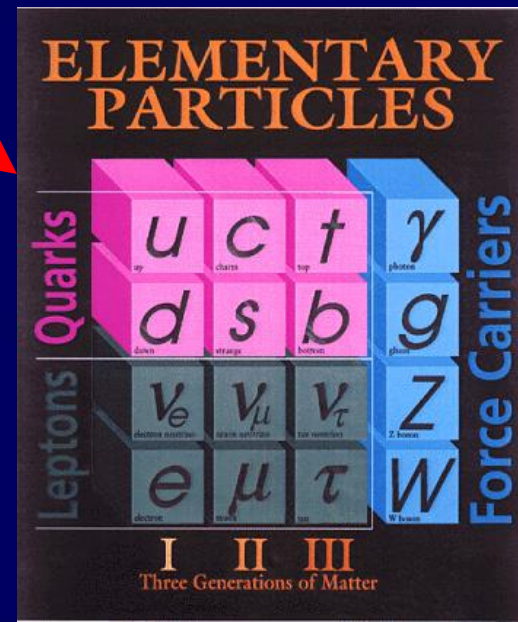
Stars and HR Diagrams

Elements are not Elementary: the Building Blocks of Nature

Group →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
↓ Period																		
1	1 H																	2 He
2	3 Li	4 Be										5 B	6 C	7 N	8 O	9 F	10 Ne	
3	11 Na	12 Mg										13 Al	14 Si	15 P	16 S	17 Cl	18 Ar	
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	57 La	* 72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 Ac	* 104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
				* 58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
				* 90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

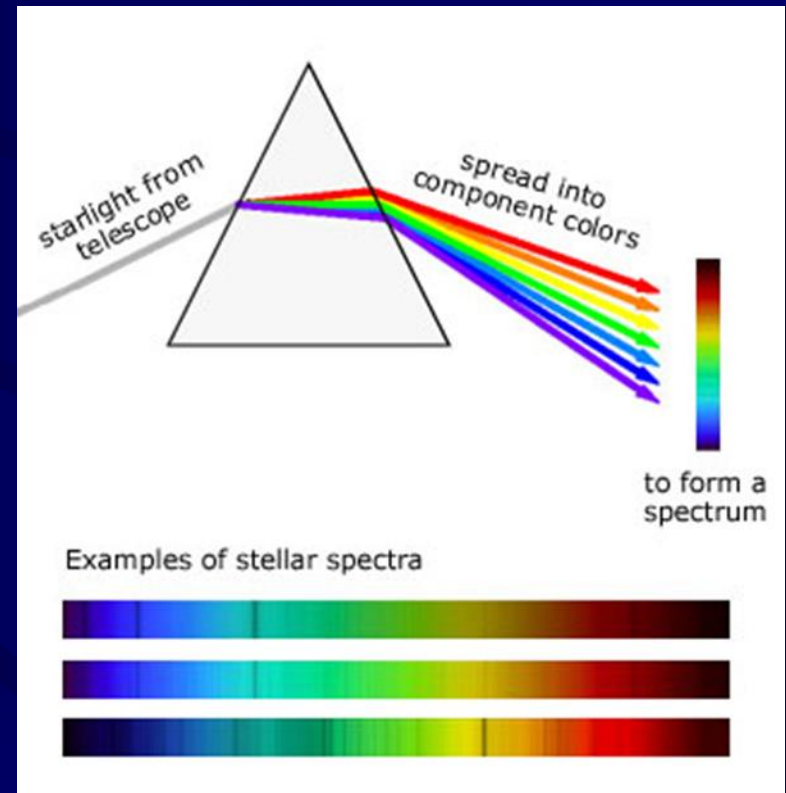


- Atoms are made from **protons**, neutrons, **electrons**
- Chemical elements are named by the **number A of protons** in their nucleus
- Atoms with same A but different **number of neutrons N** are called isotopes or nuclides



Classify Stars – to understand them!

- What properties can we measure?
 - distance
 - velocity
 - temperature
 - size
 - luminosity
 - chemical composition
 - Mass
- Which properties are useful/significant?



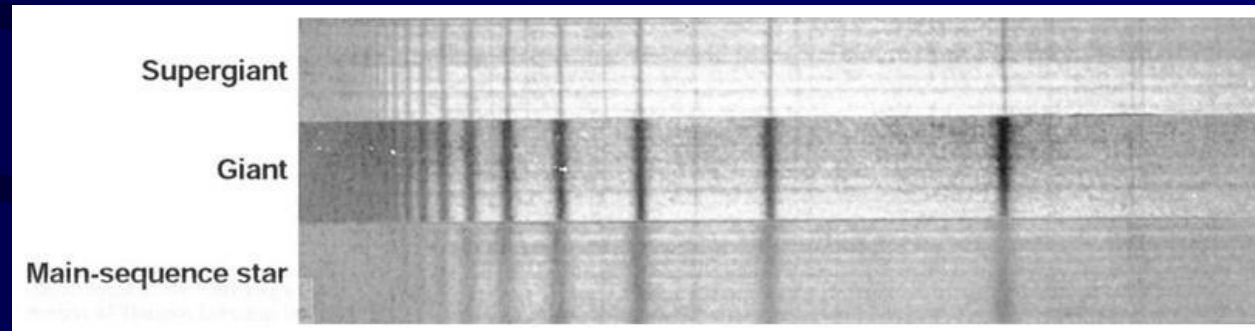
Classification of the Stars: Temperature

<u>Class</u>	<u>Temperature</u>	<u>Color</u>	<u>Examples</u>
O	30,000 K	blue	
B	20,000 K	bluish	Rigel
A	10,000 K	white	Vega, Sirius
F	8,000 K	white	Canopus
G	6,000 K	yellow	Sun, α Centauri
K	4,000 K	orange	Arcturus
M	3,000 K	red	Betelgeuse

Mnemotechnique: Oh, Be A Fine Girl/Guy, Kiss Me

Making Sense of Stellar Properties

- Lots of data → How to sort them?
 - Spectral Type
 - Temperature
 - Size
 - Mass
 - Luminosity



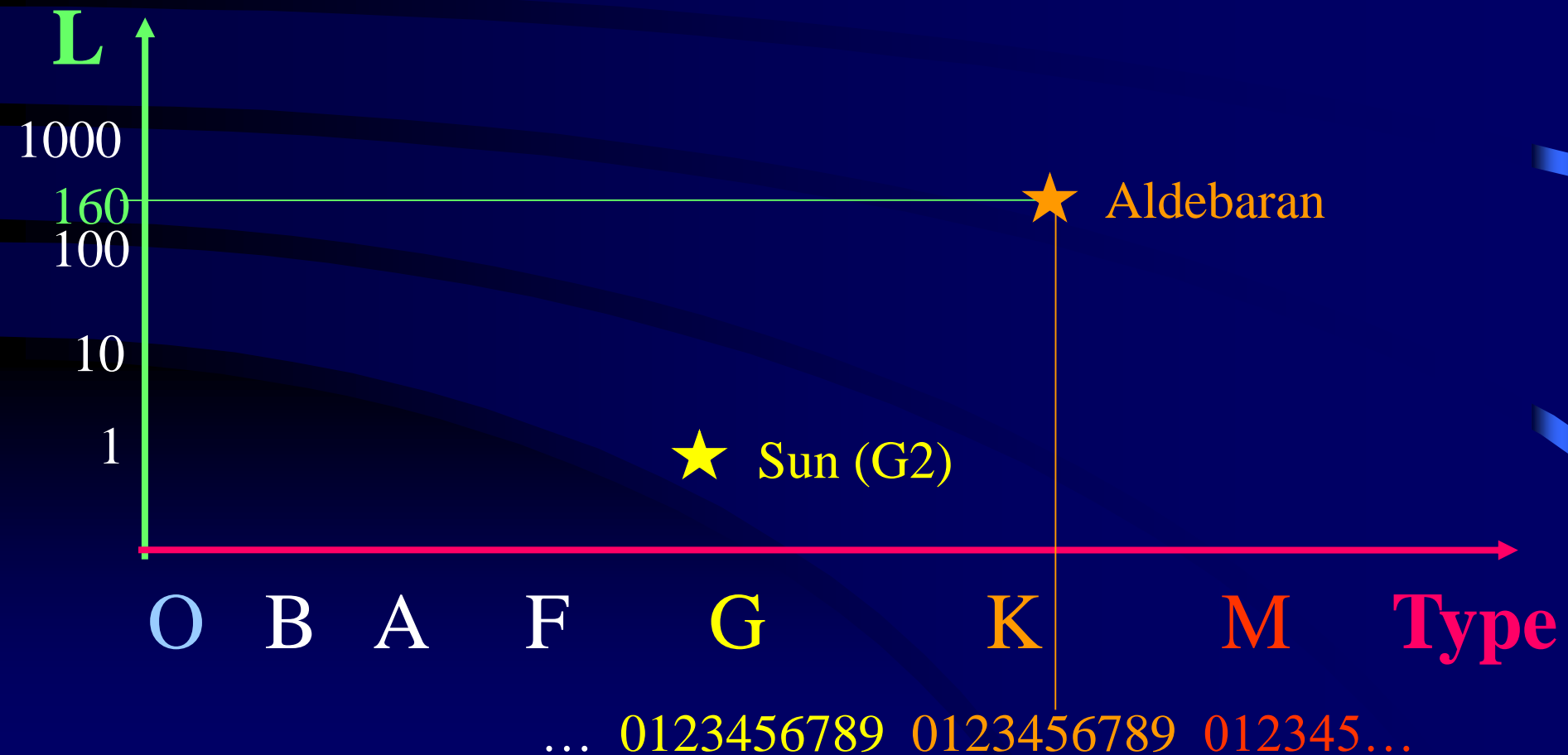
- Hertzsprung and Russell realize around 1910 that a two-dimensional classification scheme is necessary, since different versions (giants, dwarfs...) of stars of identical spectral type exist

The Key Tool to understanding Stars: the Hertzsprung-Russell diagram

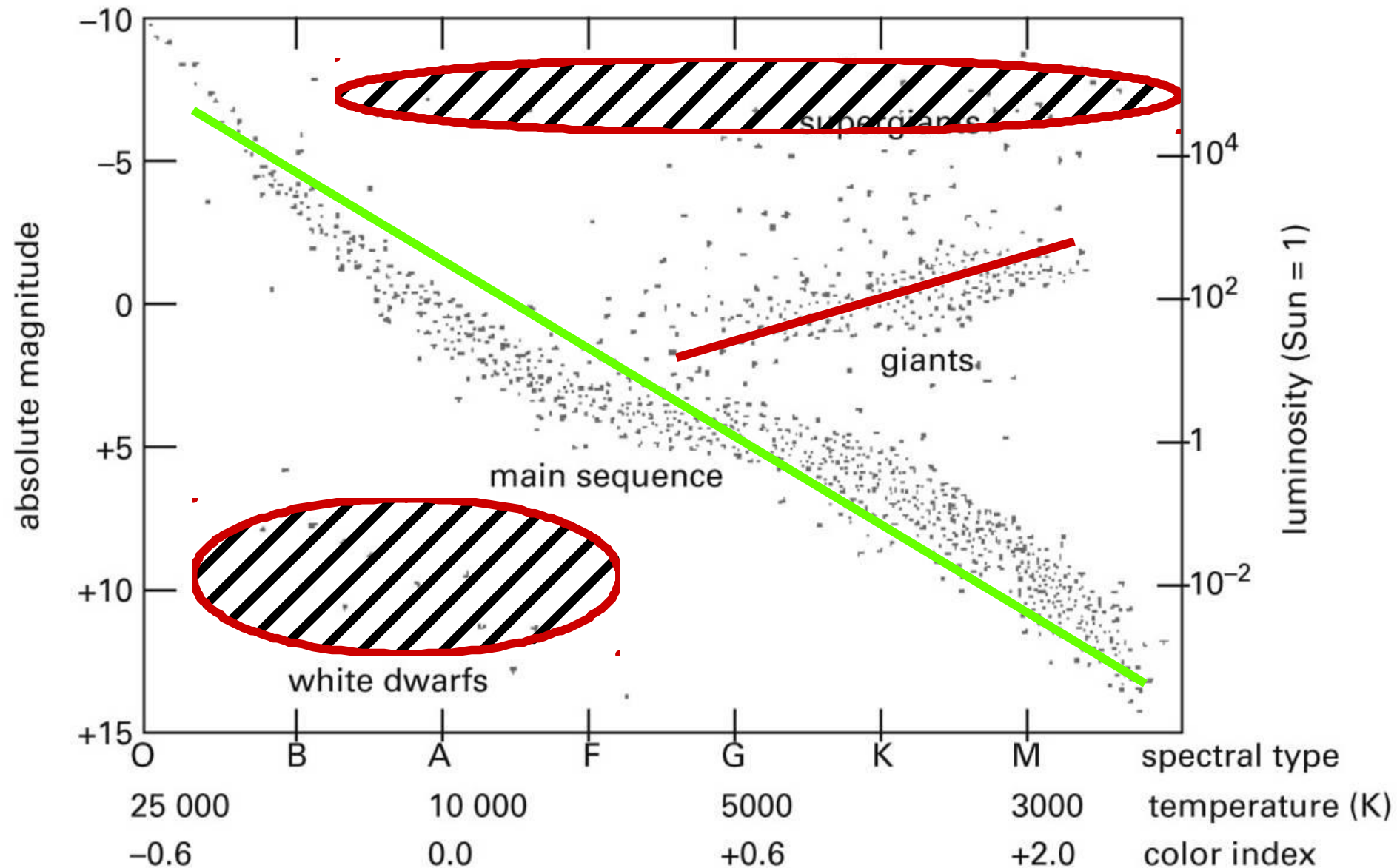
- Hertzsprung-Russell diagram is **luminosity** vs. **spectral type** (or temperature)
- To obtain a HR diagram:
 - get the luminosity. This is your **y-coordinate**.
 - Then take the spectral type as your **x-coordinate**, e.g. **K5** for Aldebaran. First letter is the spectral type: **K** (one of OBAFG**K**M), the arab number (**5**) is like a second digit to the spectral type, so K0 is very close to **G**, K9 is very close to **M**.

The Hertzsprung Russell-Diagram (HRD)

- Example: Aldebaran, spectral type K5,
luminosity = 160 times that of the Sun

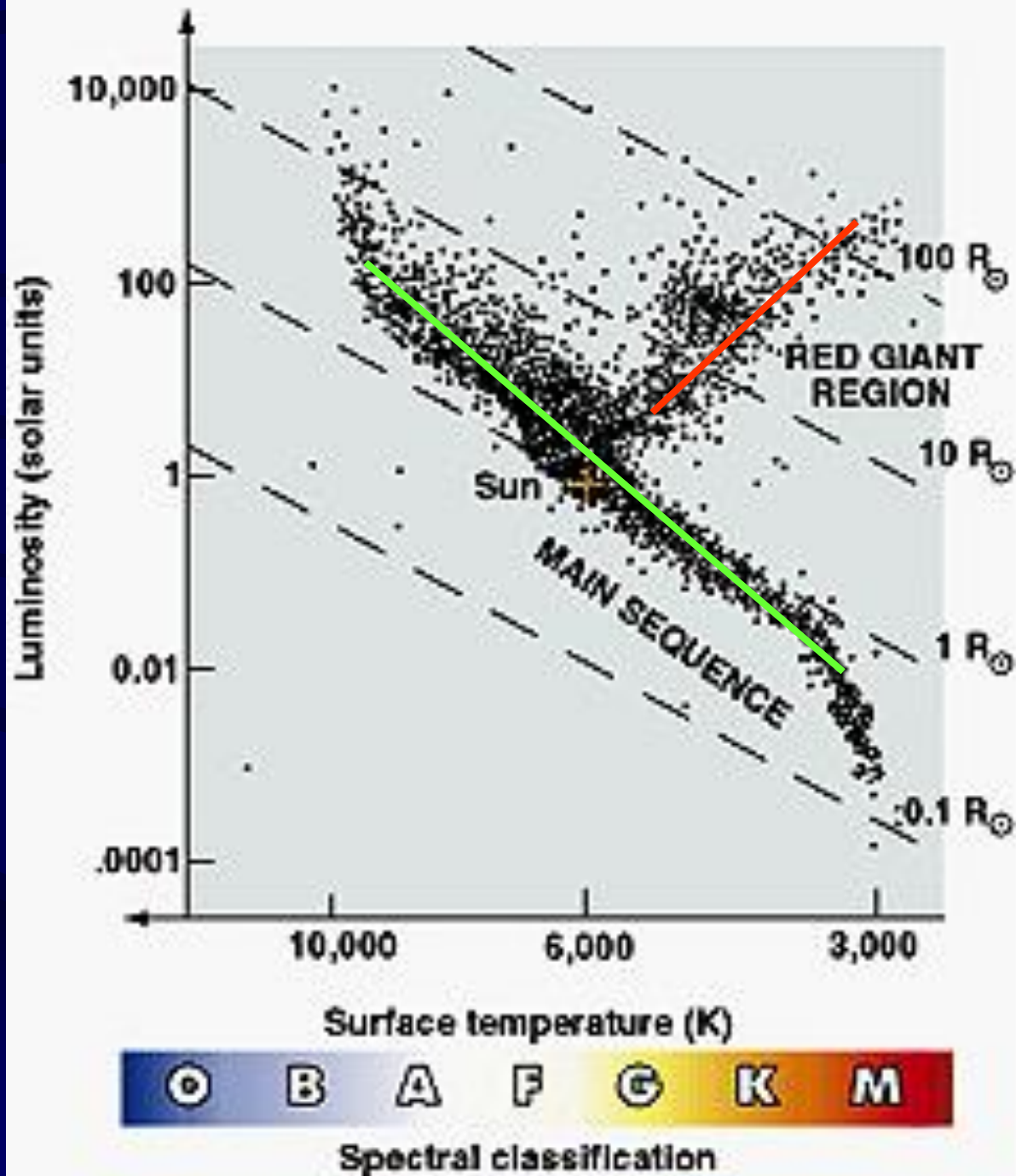


Hertzsprung-Russell Diagram



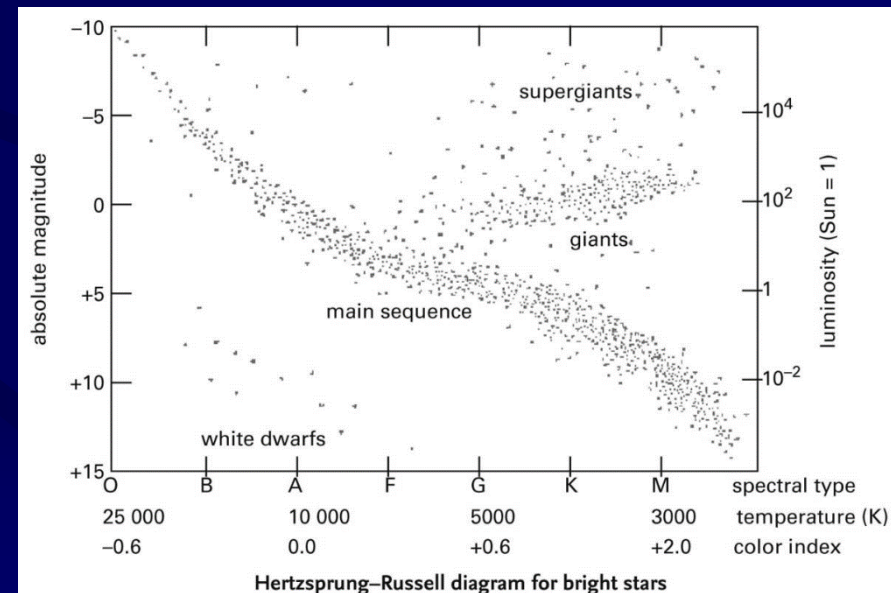
The Hertzsprung-Russell Diagram

- A plot of absolute luminosity (vertical scale) against spectral type or temperature (horizontal scale)
- Most stars (90%) lie in a band known as the Main Sequence



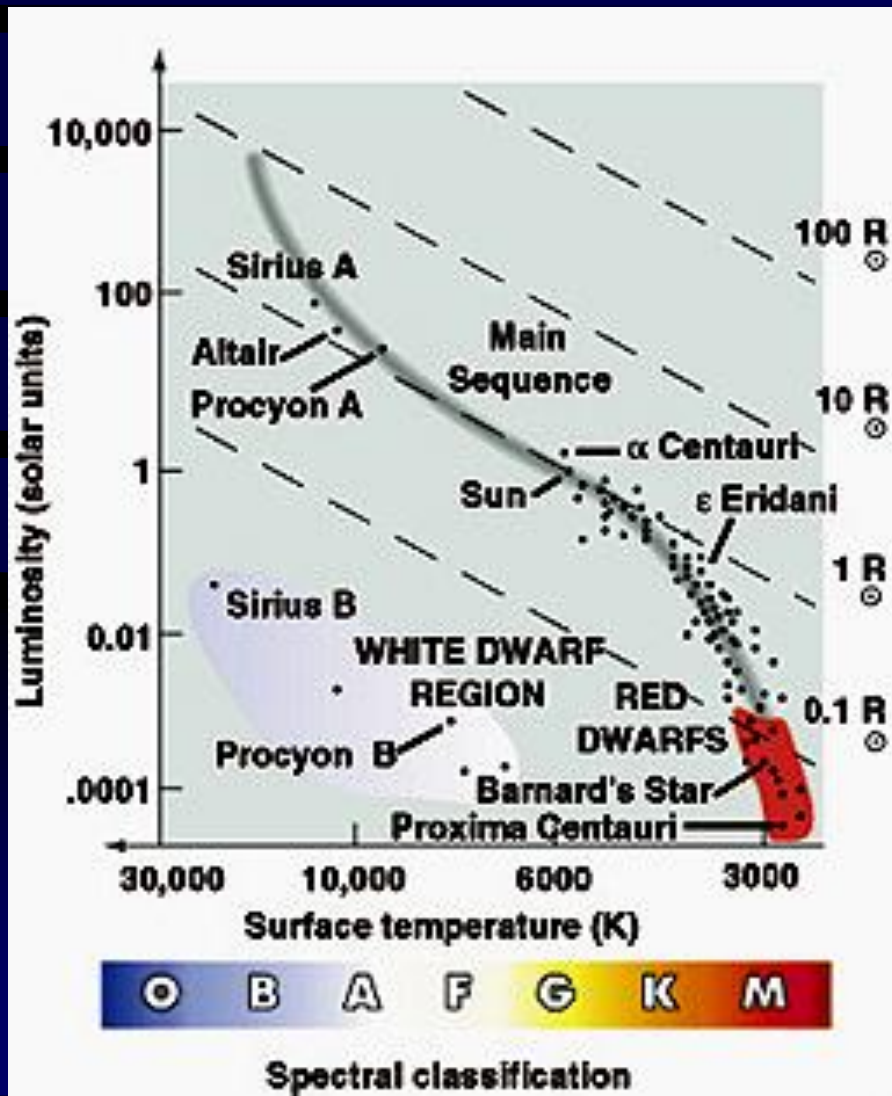
HRD: Executive Summary

- Most stars are Main Sequence stars (90%)
 - They seem “normal”, since they are the majority and obey Stefan-Boltzmann ($L=k R^2T^4$)
- Other Groups are counter-intuitive
 - Red Giants: bright yet cool
 - White Dwarfs: hot yet dim
 - Supergiants: superbright regardless of temperature

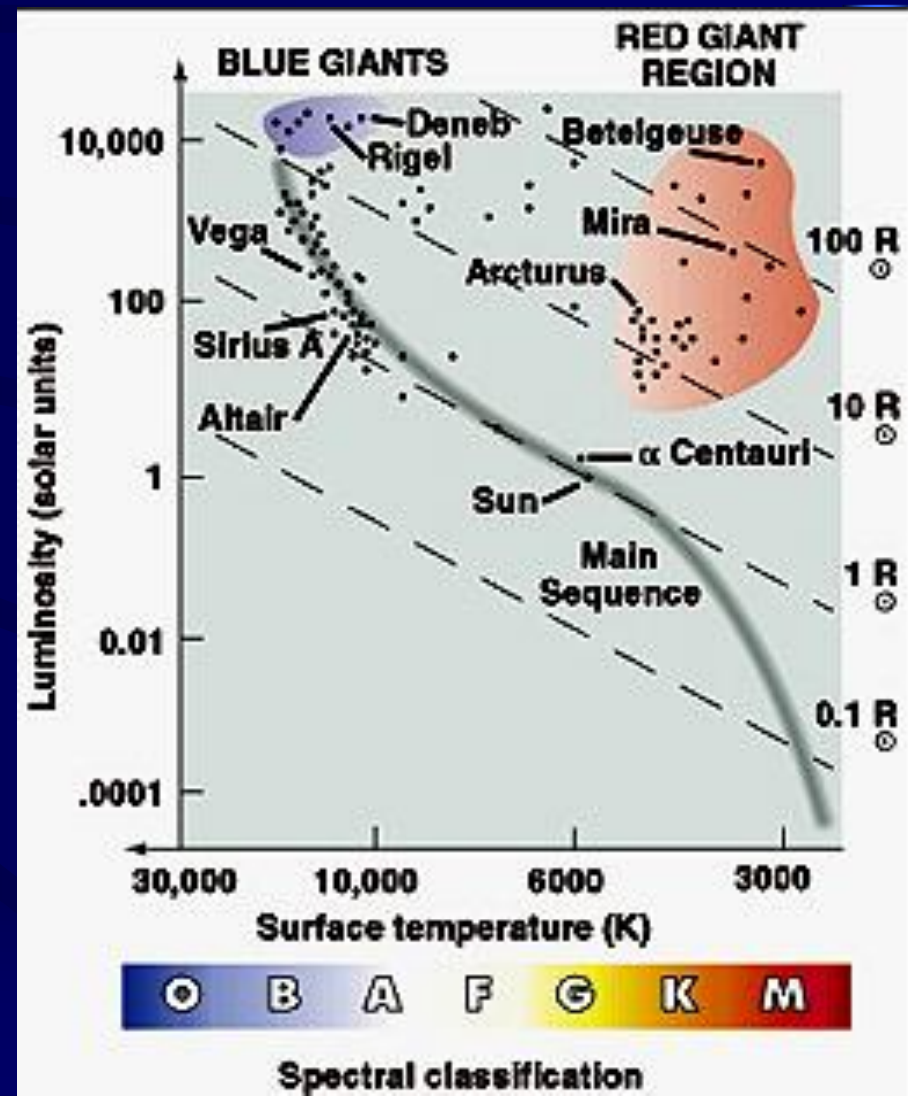


Hertzprung-Russell diagrams

... of the closest stars

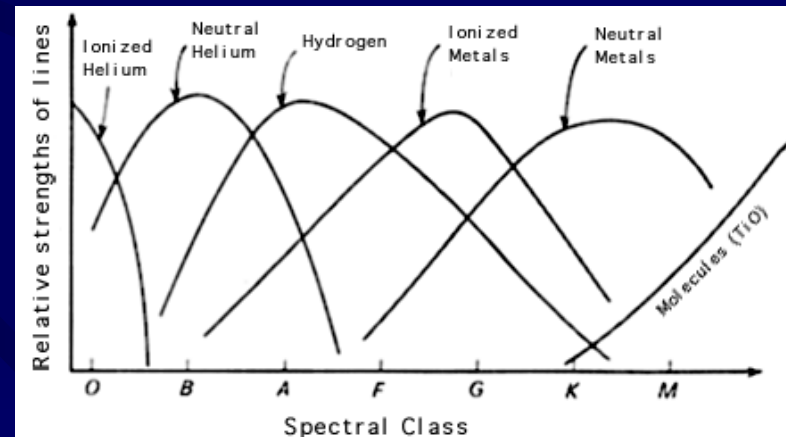


...of the brightest stars



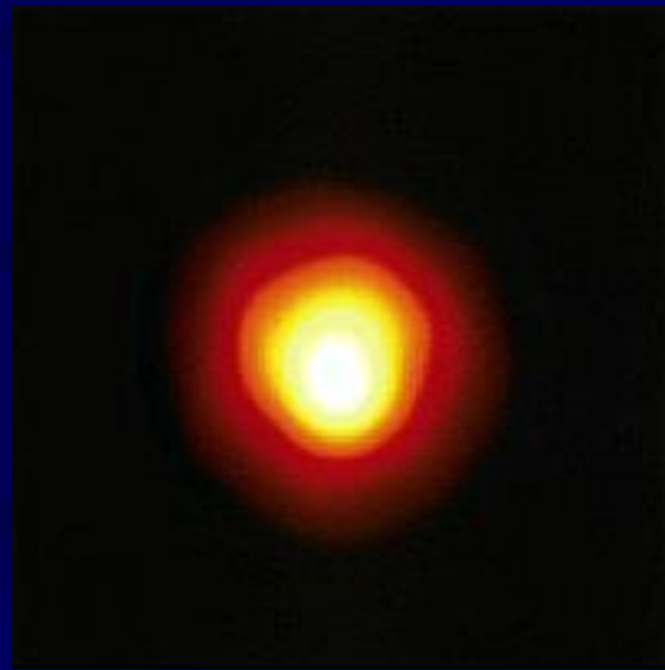
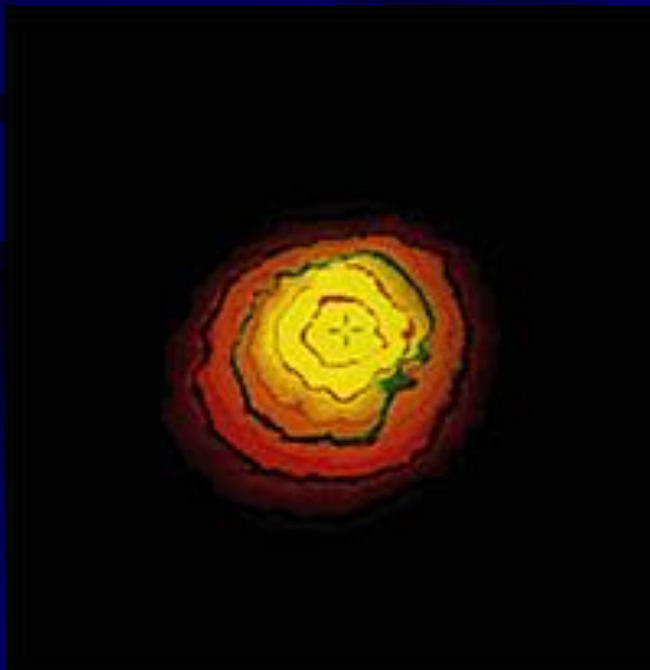
Misunderstandings

- Do we need 4 models for 4 groups?
- The redundancy of spectral type and temperature was not known until the work of Saha starting 1920
- The sun was falsely considered a “hot earth” containing “some hydrogen in its atmosphere but certainly not in its interior” until Cecilia Payne (1925)



Measuring the Sizes of Stars

- Direct measurement is possible for a few dozen relatively close, large stars
 - Angular size of the disk and known distance can be used to deduce diameter



Indirect Measurement of Sizes

- Distance and brightness can be used to find the luminosity:

$$L \propto d^2 B \quad (1)$$

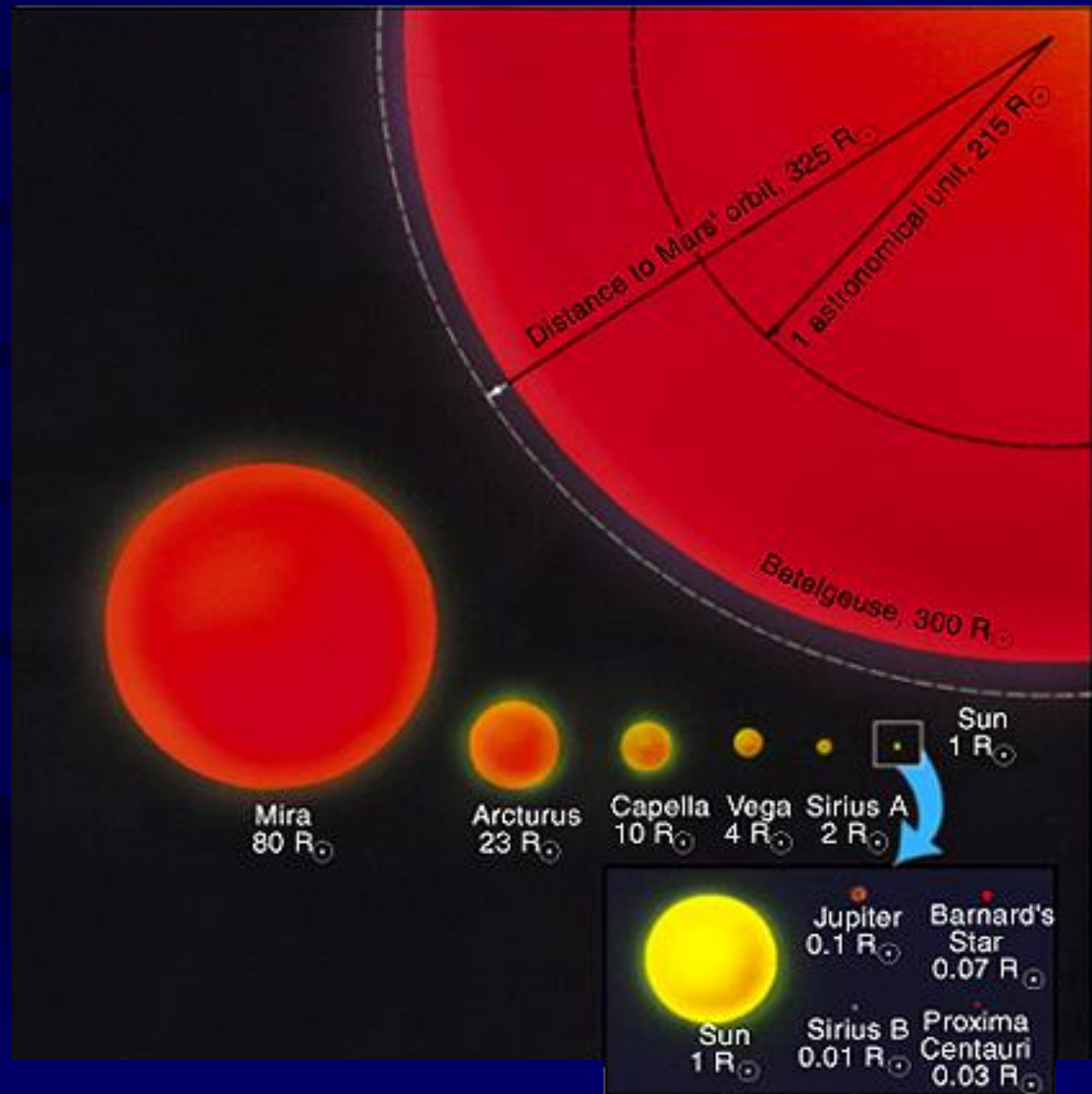
- The laws of black body radiation also tell us that amount of energy given off depends on star size and temperature:

$$L \propto R^2 \times T^4 \quad (2)$$

- We can compare two values of absolute luminosity L to get the size

Sizes of Stars

- Dwarfs
 - Comparable in size, or smaller than, the Sun
- Giants
 - Up to 100 times the size of the Sun
- Supergiants
 - Up to 1000 times the size of the Sun
- Note: Temperature changes!



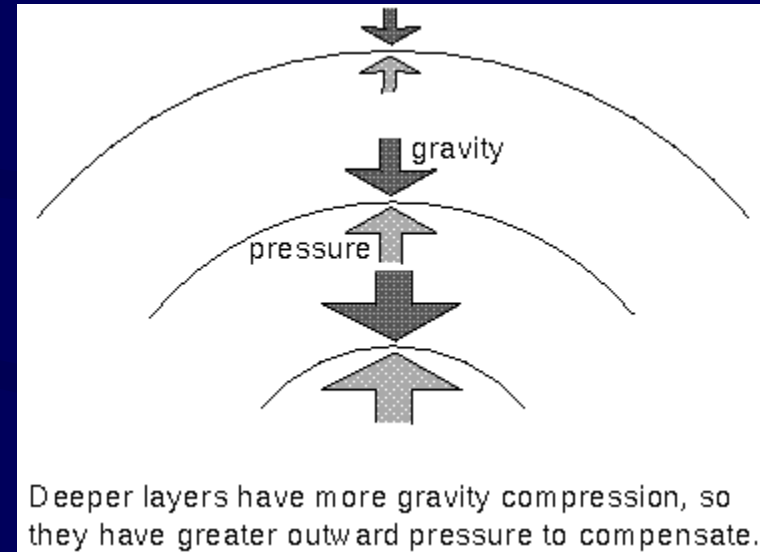
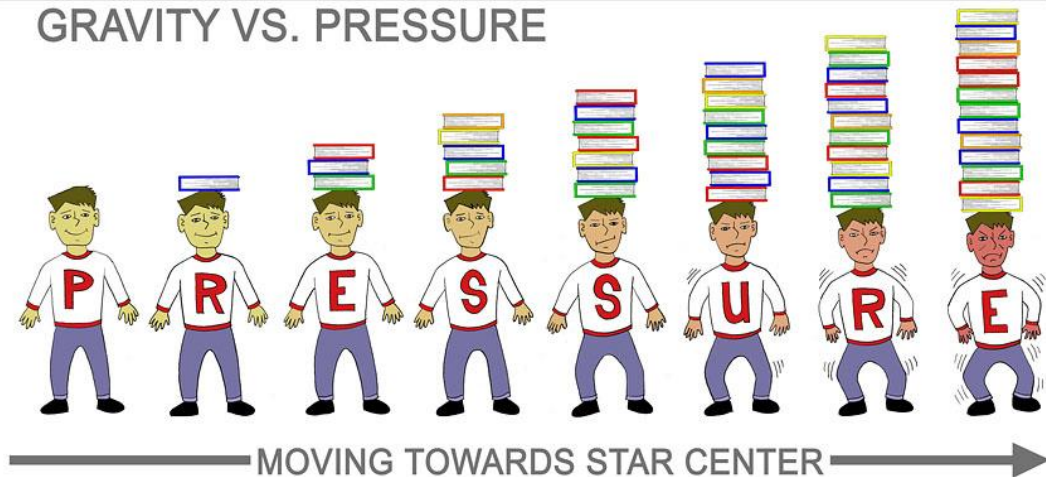
Modeling Stars



- Stars are gas(?) balls
- They produce energy (in their center?)
- To remain stable, pressure has to balance gravity
- The energy has to be transported to the surface of the star
- The star's surface will radiate according to its temperature (like a blackbody)

Gaining Intuition

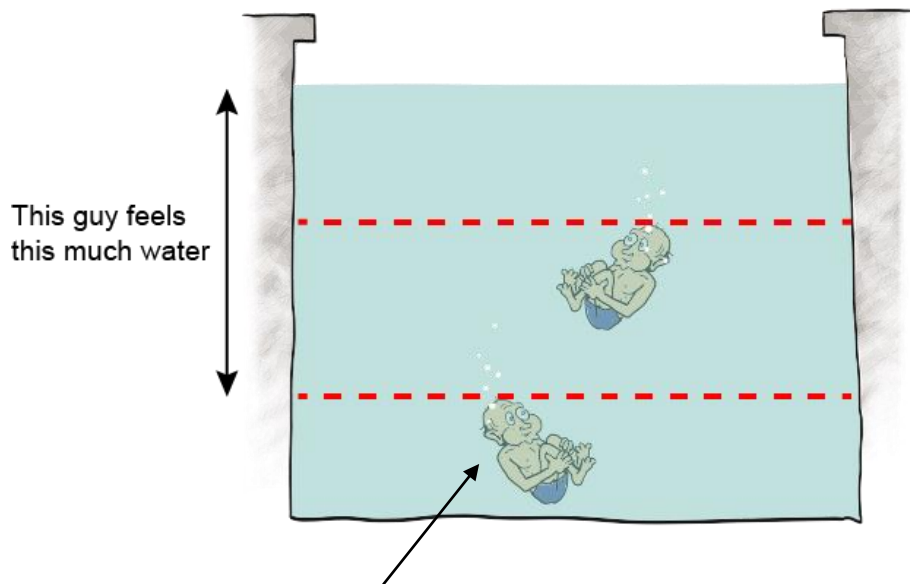
HYDROSTATIC EQUILIBRIUM IN A STAR GRAVITY VS. PRESSURE



- The “rest of the star” pushes down on a specific part of it
- The equilibrium holds for every part of the star; the forces that add up to zero are different in different parts of the star

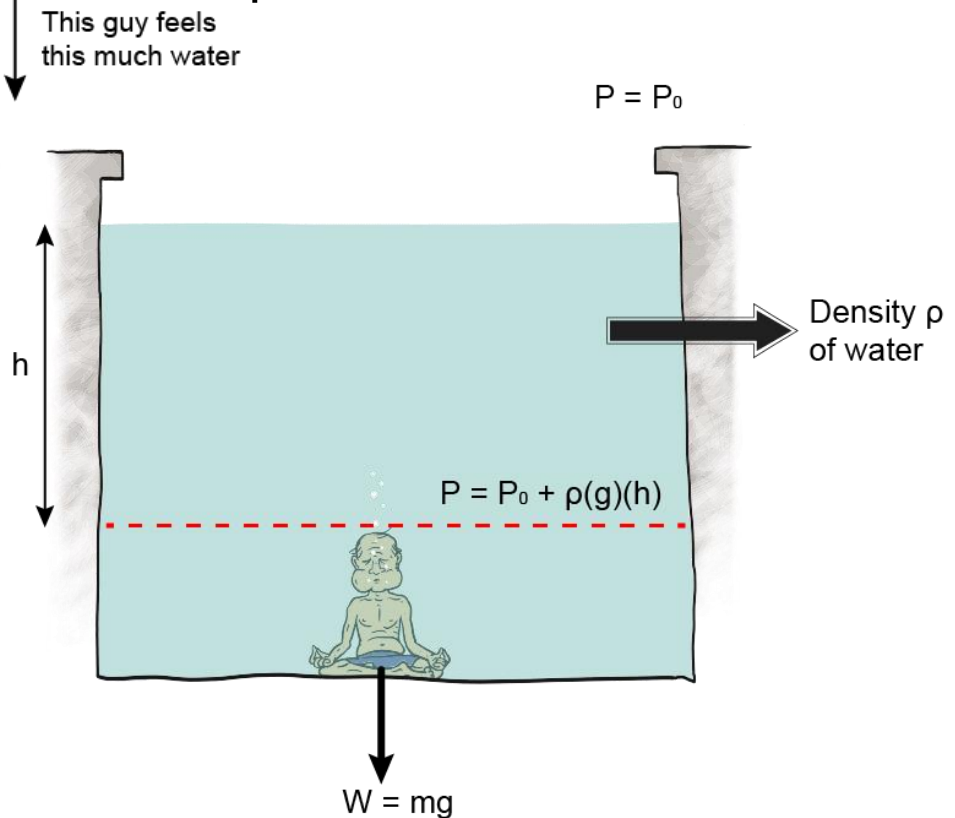
The Density $\rho(r)$ has to be right!

Fill pool with water



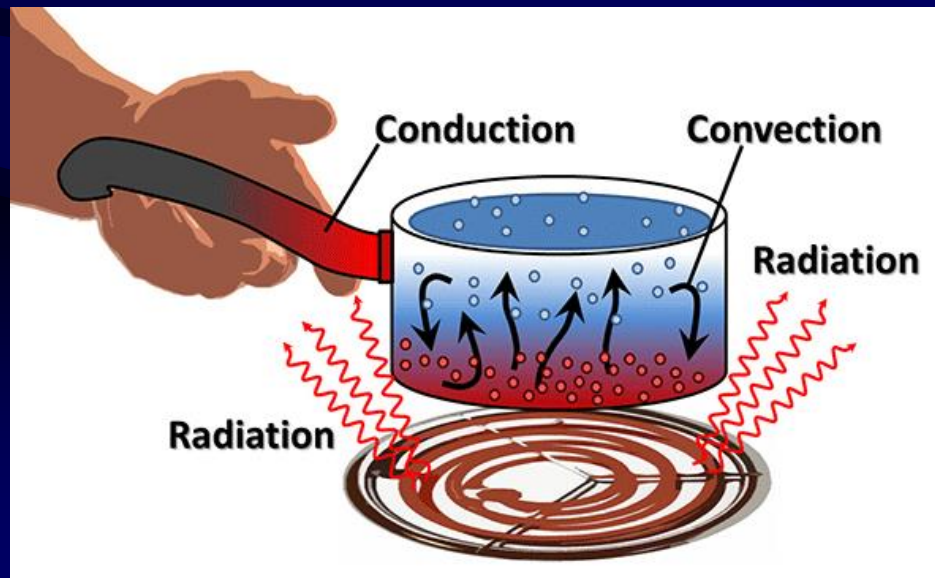
This guy sinks lower by exhaling air, i.e. increasing his density.

Fill pool with water



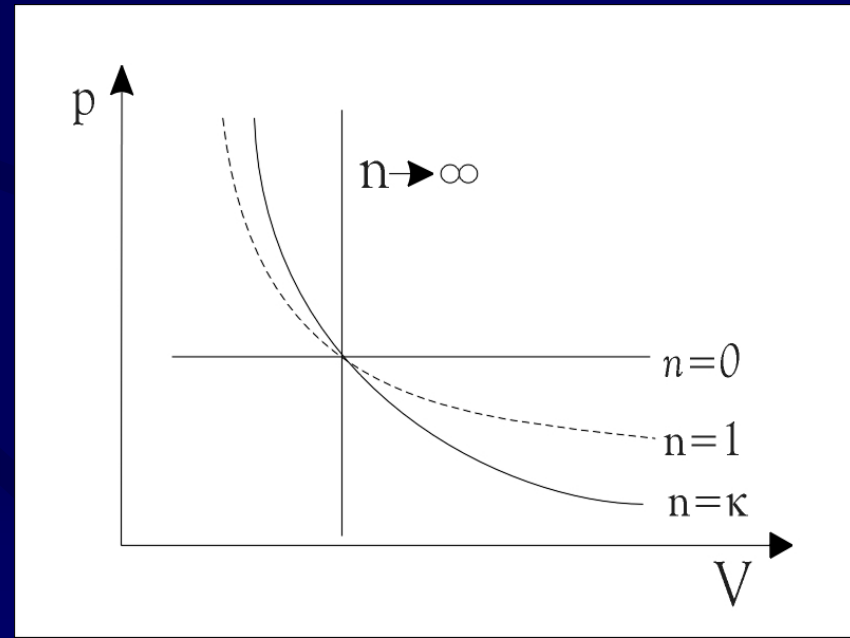
Three Mechanisms of Energy Transfer

- In stars: either **convection** or **radiation**
- Criterion: if **temperature gradient** is too steep (superadiabatic) then radiation dominates



Emden Model: Stars as Gas Balls

- Stars: Self-gravitating gas balls with a general, polytropic relation (index n) between their density and pressure: $P = K\rho^{1+1/n}$
- Temperature inversely proportional to radius
 - Hottest at center
 - Contracting \rightarrow heat up
 - Expanding \rightarrow cool down
- Densest at center



The Scientific Method at Work

- **Theory** (stellar energy production, hydrodynamics and radiative stability)
- PLUS **Observation** (stellar spectra, HRD)
- YIELDS **Stellar Model**, i.e. understanding of Stars

Filling the intellectual Vacuum: Energy Production

- Contenders:
 - Gravitational contraction
 - Radioactivity (1903)
 - Annihilation ($E=mc^2$, 1905) of proton and electron
 - Hydrogen to helium nuclear fusion
- From early 1920s: probably fusion, but how?
 - Gamov 1928: QM tunneling can overcome electrostatic repulsion of protons

Eddington: Standard model without (knowing about) energy production

- Adding a new feature: stars are actively producing energy, hence radiative energy transfer is important
- New property: opacity $\kappa(r)$ (To which degree does the stellar substance hinder radiation flow?)
- Need new equation for new unknown: thermal equilibrium: every “parcel” of the star radiates as much energy as it produces plus receives

What we want from a Stellar Model

- Density in all parts of the star: $\rho(r)$
- Temperature in all parts of the star: $T(r)$
- Pressure in all parts of the star: $P(r)$
- Opacity in all parts of the star: $\kappa(r)$
- Energy production in all parts of the star: $\epsilon(r)$

(Usually a spherical star is assumed, so r is the radial variable (distance to center))

Eddington's Standard Star Model (1926)

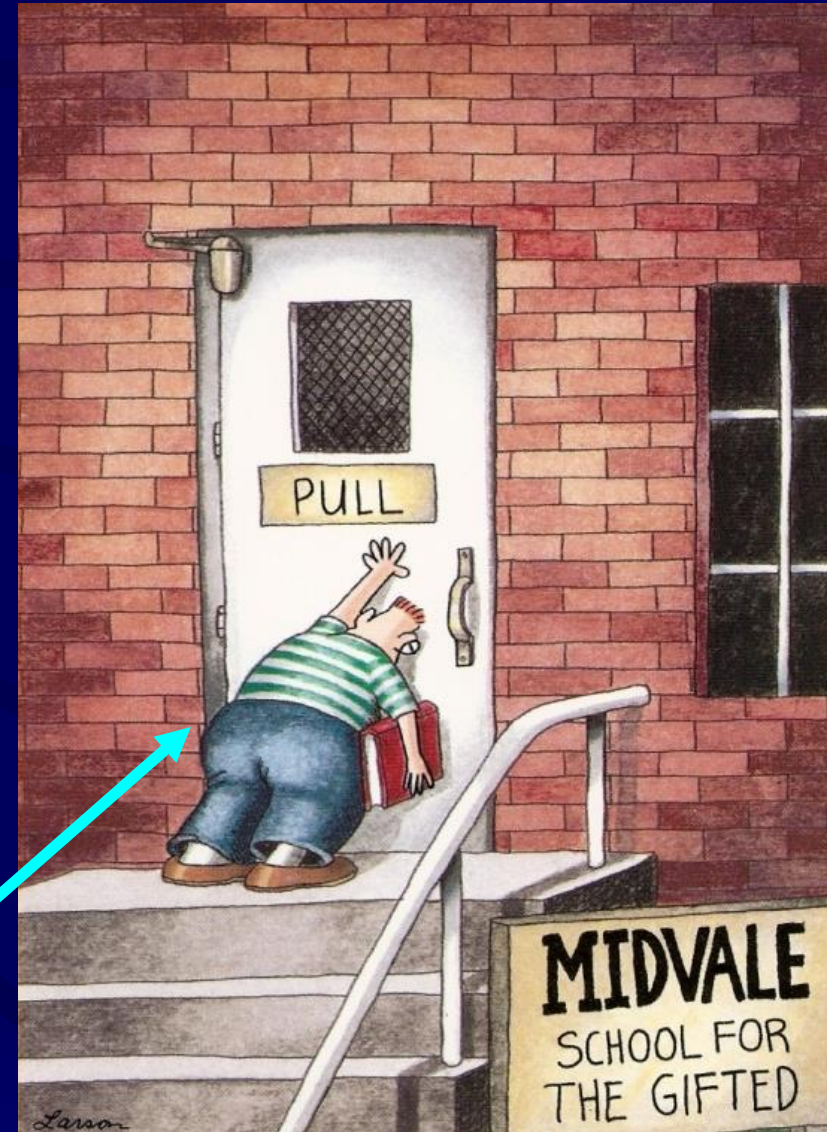
- Can recycle Emden equation since adding radiation just means that “gas ball” is a polytrope of index $n=3$ (special case of Emden's “Gaskugel”)
 - Assumption: opacity and energy function are constants
- Without knowing where the energy comes from, a realistic star model results, pointing towards the fact that the energy must be created in the core, where temperature is hot (millions K) and density is large

The Vogt-Russell theorem (1926)

- The structure of a star, in hydrostatic and thermal equilibrium with all energy derived from **nuclear reactions**, is uniquely determined by its mass and the distribution of chemical elements throughout its interior
- More than a decade before the “nuclear reactions” were specified!

Controversy: How can Stars be Gas balls?

- Too dense
 - Only liquids and solids produce continuous spectrum
 - Immense pressure at center
- (Phenomenologist Eddington
vs the mathematicians)

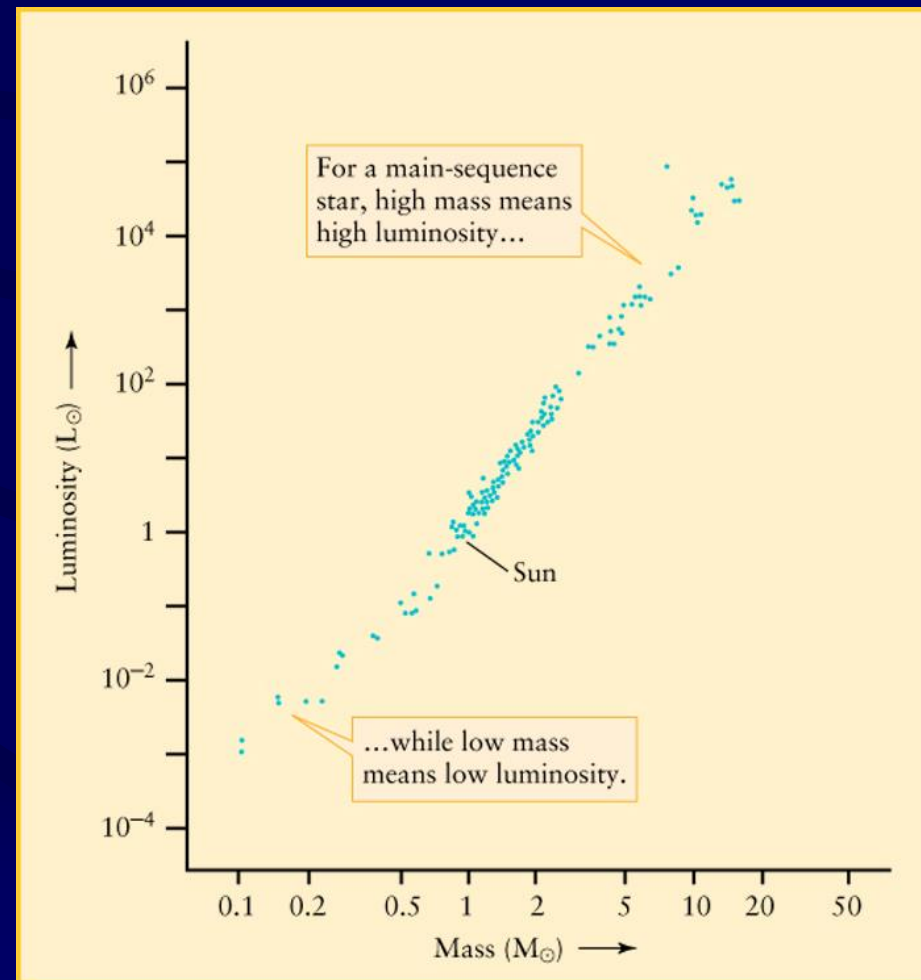


Can't argue with Success: Mass-Luminosity Relation

- Eddington was able to show that stars' luminosity basically only depend on their mass:

$$L = a M^{3.5}$$

- This killed Russell's theory of stellar evolution (blue stars cannot cool and shrink into red dwarfs with $M=\text{const.}$)



Stellar Models

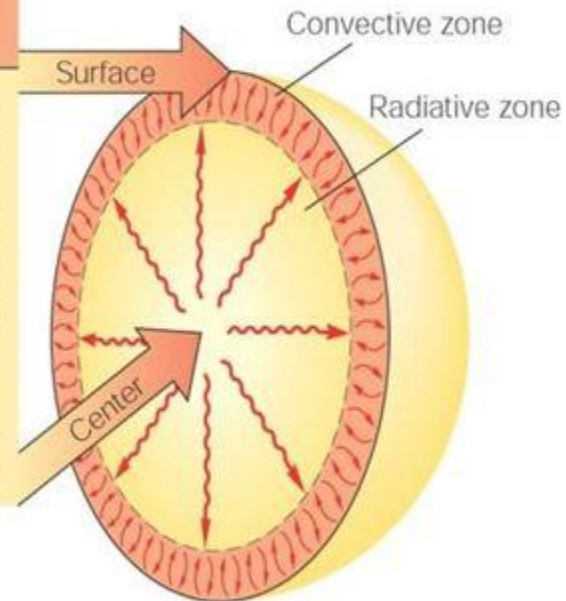


(Seeds, Backman)

The structure and evolution of a star is determined by the laws of physics

- Hydrostatic equilibrium
- Energy transport
- Conservation of mass
- Conservation of energy

R/R_{\odot}	T (10^6 K)	Density (g/cm^3)	M/M_{\odot}	L/L_{\odot}
1.00	0.006	0.00	1.00	1.00
0.90	0.60	0.009	0.999	1.00
0.80	1.2	0.035	0.996	1.00
0.70	2.3	0.12	0.990	1.00
0.60	3.1	0.40	0.97	1.00
0.50	4.9	1.3	0.92	1.00
0.40	5.1	4.1	0.82	1.00
0.30	6.9	13.	0.63	0.99
0.20	9.3	36.	0.34	0.91
0.10	13.1	89.	0.073	0.40
0.00	15.7	150.	0.000	0.00



$$\frac{dM}{dr} = 4\pi r^2 \rho$$

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon$$

$$\frac{dP}{dr} = -\frac{GM}{r^2} \rho$$

$$\frac{dT}{dr} = \frac{-3}{16\pi ac} \frac{\bar{\kappa} \rho}{T^3} \frac{L}{r^2}$$



A star's mass (and chemical composition) completely determines its properties.

That's why stars initially all line up along the main sequence.