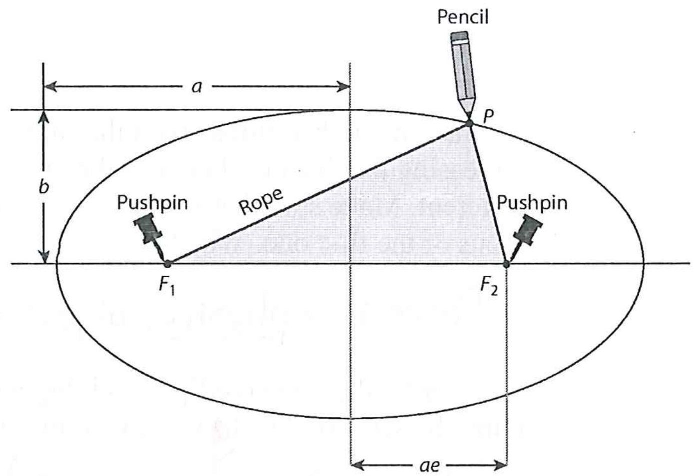
INST 2403 Activity

Kepler's Laws



The "problem of the planets" was finally solved by Johannes Kepler in 1~~9~~609 in his book *Astronomia Nova*. He had wrested the laws of planetary motion from the accurate data accumulated by Tycho Brahe. By triangulation he was able to infer the true position of the planet Mars in space, and thus constructed its orbit. This orbit turned out to be eccentric: Kepler had to use the mathematical shape of an ellipse to describe it. Thus his first law was established: **Planets move around the sun in ellipses, with the sun in one of the two foci**. The second law or equal area law then prescribes a rule as to how the planets move along this orbit: **The line connecting planet and sun moves so that it sweeps out equal areas in equal times**. An immediate consequence is the fact that planets move faster if they are closer to the sun. Note that Kepler did not explain **why** planets move faster (Newton achieved this over half a century later) – he just stated **how** they move. But that's all astronomers needed to know to compute and predict the position of the planets in their orbits and in the sky for any point in time – be it past, future or present.

I. First Law

1. Draw an ellipse on a piece of blank paper using loop of string, 2 pushpins and a pencil. Mark the position of sun in one of the focal points, i.e. at the location of one of the pushpins. ✓

2. Draw in the *major axis*, which connects the *perihelion* (closest point to the sun [Greek: *Helios*]) and the *aphelion* (farthest point from the sun). This is called the *line of the apsides.* Its orientation in space is one of the six *orbital elements* which specify the orbit.

3. The two numbers that specify an ellipse are its *semi-major axis a* and its *eccentricity e*. As the name suggests, the semi-major axis is half the major axis.

Measure it: $a =$ 13.5 cm.

Clearly, the eccentricity tells us how eccentric an ellipse is. A circle is not eccentric at all, and therefore has eccentricity e = 0. To determine the eccentricity of your ellipse, measure the distance between the two foci. Geometry tells us that this distance is equal to $d_{ff} = 2\,ea$, so

$$e = d_{ff}/(2a) = 0.68 = \frac{18.4\text{ cm}}{2\times 13.5\text{ cm}}$$

4. What is the average distance of the planet from the sun?

The semi-major axis : a, since

$r_{min} = a - ae = a(1-e)$

$r_{max} = a + ae = a(1+e)$

5. Draw in another ellipse with the same string loop, but change the eccentricity by increasing the distance between the pushpins. Draw it such that the lines of apsides is different. Make sure that one of the foci of the second ellipse coincided with the sun-focus of the first one. Why?

There is a physical object at the common focus: the sun.

6. Describe the second ellipse: Is it bigger/smaller? Is it less or more eccentric? Do you think the second planet goes around the sun in less or more time than the first one?

Ellipse 2 is less eccentric and smaller.
Since the avg distance from the sun is less ($a_2 < a_1$) it will go around the sun in less time.

II. Second Law

7. On a new sheet of paper, draw another, fairly eccentric ellipse. Draw in the line of apsides and mark perihelion and aphelion by committing the sun to one of the two foci.

✓

8. Mark off about 16 intervals of equal distance around the perimeter of the ellipse by halving the distance between the perihelion (aphelion) and the point halfway between peri- and aphelion. Then half these distances again. ✓

9. How does the planet move along the orbit? Will it take equal time to move the equal distances between the tick marks? Why or why not?

No, it moves such that the planet-sun line sweeps out equal areas in equal times ⇒ different distances in equal times

10. Shade an area in the ellipse that is bounded by the sun, the aphelion and one of the tick marks closest to it. ✓

11. Now try to find an area of equal size around the perihelion. How many tick marks do you have to include to make the two areas equal?

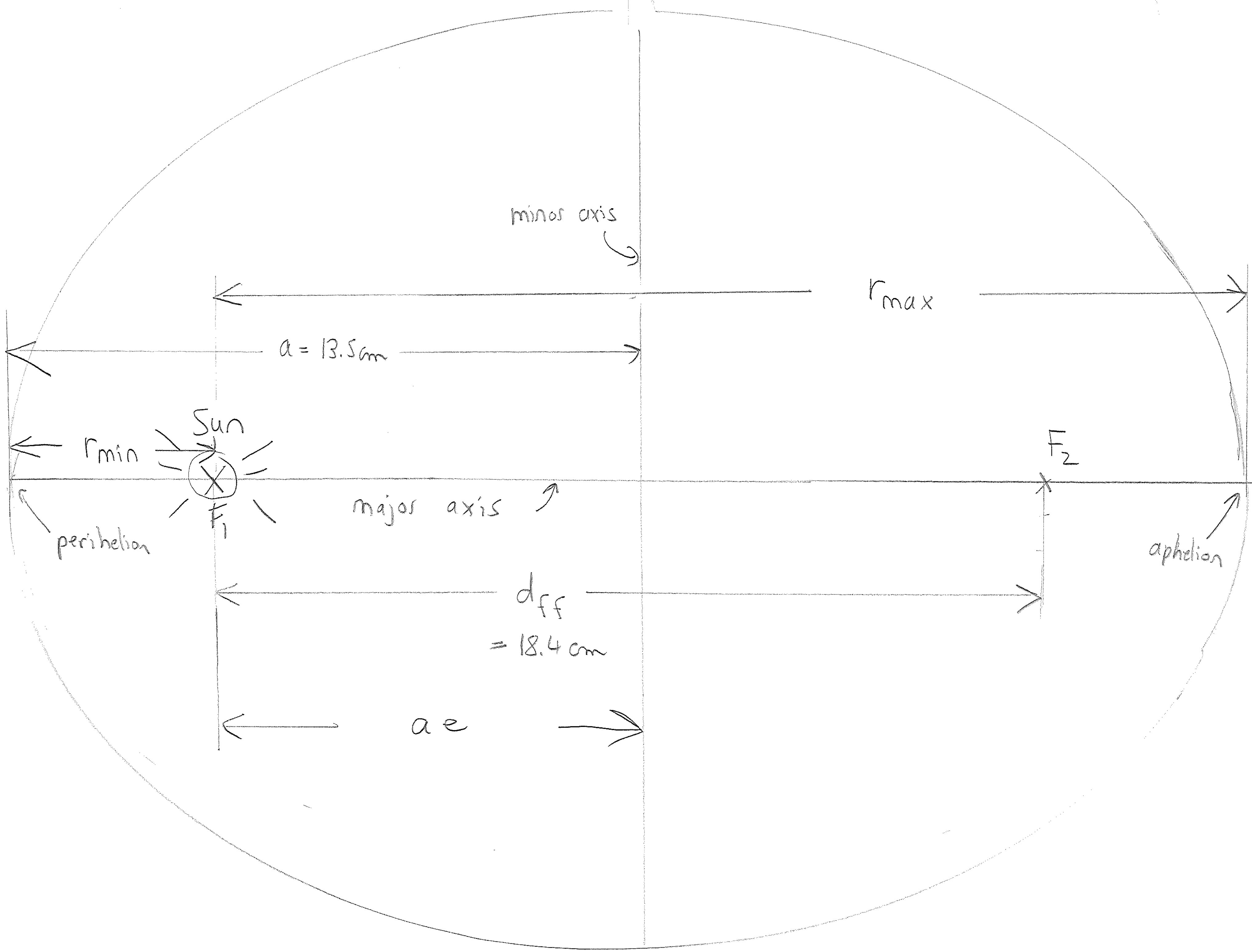
More than 4!

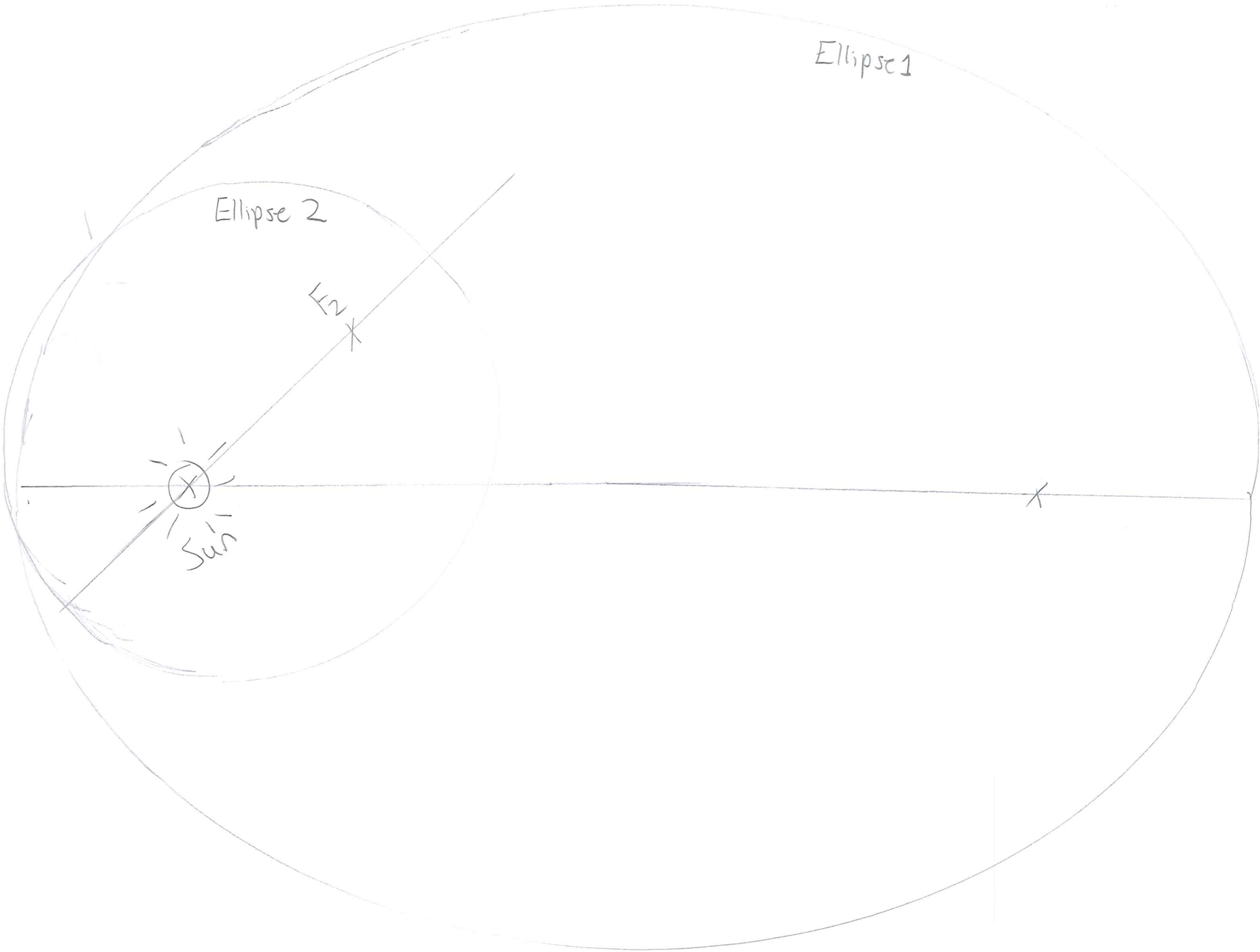
12. Kepler's second law tells us that the planet-sun line sweeps out equal areas in equal times. Does the planet therefore move equal distances in equal time? Why or why not?

No, see above. The line sun-planet is much shorter at perihelion, so the planet has to move a longer distance around its orbit to sweep out the same area.

13. How much faster does your planet move when it is closest to the sun than when it is farthest from the sun?

At least 4x as fast!


minor axis
r max
a = 13.5cm
Sun
r min
F2
major axis
F1
perihelion
aphelion
d ff
= 18.4 cm
a e


Ellipse 1
Ellipse 2
F_2
Sun

Area of perihelion segment $\approx \frac{1}{2} r_{min} \times \text{height}$

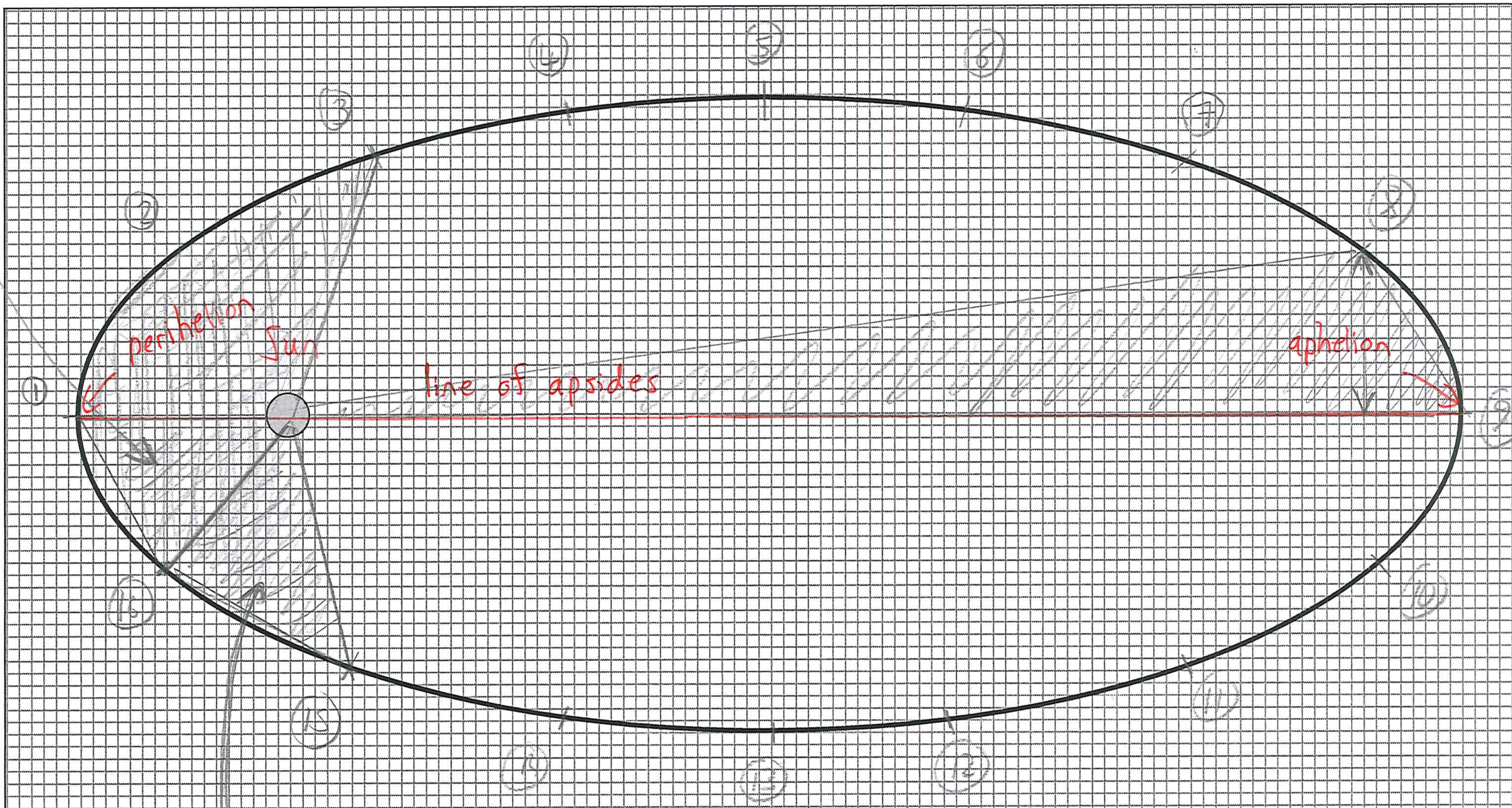
$= \frac{1}{2}(3.5\,cm) \times (3\,cm)$

$\approx 2.6\,cm^2 \rightarrow$ almost 12x smaller

Area of aphelion segment $\approx \frac{1}{2} r_{max} \times \text{height}$

$= \frac{1}{2}\, 20\,cm \times 3\,cm$

$= 30\,cm^2$



area $\approx \frac{1}{2}(3.7\,cm) \times (3.2\,cm)$

$\approx 5.9\,cm^2$

$\Rightarrow$ "4 ticks": $(2.6 + 2.6 + 5.9 + 5.9)\,cm^2 = 17\,cm^2$

$\Rightarrow$ Need more than "4 ticks"!