Formal Language Theory Practice Exercises
COMP 2230
Spring 2024
prof. stucki…

1) For each of the following regular expressions e, give the state diagram of a DFA that accepts the language $L(e)$.

(a) $(ab)^*ba$

(b) $(ab)^*(ba)^*$

(c) $((aa)^+bb)^*$

2) Construct a DFA that accepts the language:

$$L = \{w \in \{a,b\}^* \mid w \text{ contains } aba \text{ exactly once}\}$$

3) Let $\Sigma = \{a,b\}$. Show that the following languages are regular:

$$L_4 = \{w \in \Sigma^* \mid w \text{ begins with } b \text{ and } |w|_a \text{ is even}\}$$

$$L_5 = \{w \in \Sigma^* \mid w \text{ begins with } b \text{ and } |w|_b \text{ is even}\}$$

4) Let $\Sigma = \{a,b,c\}$. We say that a word w *contains a word* u if there are $v_1, v_2 \in \Sigma^*$ such that $w = v_1 u v_2$. Show that the following languages are regular:

$$L_6 = \{w \in \Sigma^* \mid |w| \text{ is even}\}$$

$$L_7 = \{w \in \Sigma^* \mid |w| \text{ is odd}\}$$

$$L_8 = \{w \in \Sigma^* \mid w \text{ contains } aa\}$$

$$L_9 = \{w \in \Sigma^* \mid w \text{ does not contain } aa\}$$

5) (Recall the last question). Let $\Sigma = \{a,b\}$ and let

$$L_1 = \{w \in \Sigma^* \mid |w| \text{ is odd}\}$$

$$L_2 = \{w \in \Sigma^* \mid w \text{ does not contain } aa\}$$

Construct a DFA that accepts $L_1 \cap L_2$ and use it to find a regular expression e such that $\mathcal{L}(e) = L_1 \cap L_2$.

6) Give a context-free grammar (V, Σ, P, S) that generates the set of all regular expressions over $\{a, b\}$. (Note that $\Sigma \neq \{a, b\}$.)

7) For each of the following languages give a context-free grammar that generates the language:

$$
\begin{aligned}
L_1 &= \{a^n b^k a^m \mid k = n + m,\ n, k, m \geq 0\} \\
L_2 &= \{w \in \{a, b\}^* \mid |w|_a \text{ is even}\} \\
L_3 &= L((ab)^*(a + bb)^*) \\
L_4 &= \{w \in \{a, b\}^* \mid |w| = 2k + 1 \text{ and } w_1 = w_{k+1}\}
\end{aligned}
$$

where w_i denotes the i-th symbol in a word w. That is, L_4 consists of all words of odd length that have the same symbol in the first and middle positions.

8) For each of the following languages L_i, construct a PDA $\mathbf{M}_i$ that accepts L_i, $i = 1, 2, 3, 4$.

- $L_1 = \{a^n b^{2n} \in \{a, b\}^* \mid n \geq 0\}$.
  Give a successful computation on $aabbbb$.
- $L_2 = \{w \in \{a, b\}^* \mid w = w^R\}$.
  Give successful computations on $baab$ and $abbba$.
- $L_3 = \{w \in \{a, b\}^* \mid |w|_a = |w|_b\}$.
  Give a successful computation on $abbaab$.
- $L_4 = \{w \in \{a, b\}^* \mid |w|_a \leq |w|_b\}$.
  Give a successful computation on $abbbaab$ and an unsuccessful computation on aba.