

We present a hybrid version of the Polya and Schoenfeld schemes.

I. Understand the Problem (Analysis)

1. Identify the unknown.
2. Isolate the hypotheses and the data.
3. Develop a representation of the problem.
 - a. Draw a figure.
 - b. Introduce suitable notation.
4. Examine special cases.
 - a. Select special values to acquire a "feel" for the problem.
 - b. Consider extreme cases.
 - c. Evaluate integer parameters at $n = 1, 2, 3, \dots$ and look for a pattern
5. Simplify the problem.
 - a. Exploit symmetry.
 - b. Choose appropriate units.

II. Devise a Plan (Exploration)

1. Consider essentially equivalent problems.
 - a. Replace conditions by equivalent conditions.
 - b. Recombine the elements of the problem in various ways.
 - c. Introduce auxiliary elements.
 - d. Reformulate the problem.
 - i. Change perspective or notation.
 - ii. Argue by contradiction or contrapositive.
 - iii. Take the problem as solved; i.e., assume you have a solution and determine its properties.
2. Consider slightly modified problems.
 - a. Aim for subgoals.
 - b. Relax a condition, then restore it.
 - c. Consider case analysis.
3. Consider broadly modified problems.
 - a. Construct an analogous problem with fewer variables.
 - b. Generalize the problem.
 - c. Hold all but one variable fixed to determine that variable's impact.
 - d. Try to exploit any problem that has a similar form, hypothesis, or conclusion.

III. Carry out the Plan (Verification)

1. Check each step.
2. Prove that each step is correct.

IV. Look Back (Verification)

1. Apply these specific tests to your solution:
 - a. Does it use all the pertinent data?
 - b. Does it conform to reasonable estimates or predictions?
 - c. Does it withstand tests of symmetry, dimension analysis, or scaling?
2. Apply these general tests:
 - a. Can the result be obtained differently?
 - b. Can the result be verified in special cases?
 - c. Can the result be reduced to known results?
 - d. Can the result be used to derive other known results?

The remainder of this unit will be devoted to comments on certain aspects of this outline. More details can be found in Polya's books *How to Solve It* [6], *Mathematical Discovery* [7], and *Mathematics and Plausible Reasoning* [8] and in Schoenfeld's *Mathematical Problem Solving* [9].